Tel Aviv University

The Lester & Sally Entin Faculty of Humanities

The Shirley & Leslie Porter School of Cultural Studies

Exhaustivity

A Semantic Account of 'Quantity' Implicatures

THESIS SUBMITTED FOR THE DEGREE "DOCTOR OF PHILOSOPHY"

by

Aldo Sevi

SUBMITTED TO THE SENATE OF TEL AVIV UNIVERSITY

March 2005

This work was carried out under the supervision of

Prof. Fred Landman

Acknowledgments

This dissertation has roots in two seminars taught by my advisor, Fred Landman some years ago. One – on the meaning of *only*, the other – on the adjectival theory of indefinite predicates and arguments. Since my early days in the linguistics department at Tel Aviv University, I knew that if I were to write a Ph.D. dissertation, it would be (against my better judgment) about implicatures. But who could have guessed this would involve so many λ's…

First and foremost I would like to express my gratitude to my advisor Fred Landman, who kept faith in me even when I lost it. Under his patient, kind, confident and encouraging supervision, I managed to develop, slowly but surely, a crude idea (which barely held a few pages) to the bulk of this thesis.

I thank Nirit Kadmon for her very helpful comments on an early draft of chapter 1, and for our discussions on the ideas developed in this dissertation. I will always be grateful to Nirit for introducing me to the field of semantics.

I'm grateful to Alex Grosu and to Fred Landman for generous financial support during the years I've been their research assistant on their joint project, and on Alex Grosu's projects, funded by the Israeli Academy of Sciences and Humanities.

Many thanks to the members of the semantics/pragmatics student group at Tel Aviv University: Vita Barabash, Ido Ben-Tzvi, Uriel Cohen Priva, Tal Kedar, Dima Levinson, Galit Sassoon, Aynat Rubinstein, Miki Zar, and to Mira Ariel (who honored us by participating in our meetings). They've been my first audience at early stages of this work. I thank them for listening, commenting, and encouraging. I also thank Mira Ariel for discussing with me some aspects of this dissertation in its final stages.

I thank the audiences at the departmental colloquia of Ben Gurion University of the Negev, Tel Aviv University, Bar-Ilan University, and the Hebrew University of Jerusalem to whom I presented earlier versions of some aspects of this dissertation.

During my long years as an MA and PhD student at Tel Aviv University I made some good friends among my fellow students. I thank Ziva Wijler, Nurit Assayag, Shai Cohen, Galit Sassoon and Michael Becker for being such a wonderful company.

I would like to mention my students in the courses I taught over the years at Tel Aviv and Jerusalem. The experience taught me a lot, and I hope I managed to inspire them half as much as my semantics teachers, Fred Landman and Nirit Kadmon have inspired me.

I'm grateful to my parents, Rosa and Eli Sevi, who love and support me unconditionally. They were even more anxious than me to see this dissertation completed. So, finally, here it is!

This dissertation is dedicated to Eliezer Zilberberg, my love, and best friend. Without Eliezer’s infinite love, encouragement and confidence, I wouldn’t have made it.

Abstract

This dissertation is concerned with inferences which are traditionally attributed to Grice's first maxim of Quantity (in particular, scalar and clausal implicatures), like the inference from (1) to (2), (3) and (4), and from (5) to (6).

(1) Mary has a snake or an iguana

(2) Mary doesn't have a snake and an iguana

(3) As far as the speaker knows, it is possible that Mary has a snake, and it is possible that Mary doesn't have a snake.

(4) As far as the speaker knows, it is possible that Mary has an iguana, and it is possible that Mary doesn't have an iguana.

(5) A: Do you have any juice?
B: I have orange and grapefruit

(6) B doesn't have apple/pear/peach/etc. juice

I start out by arguing that the original Gricean approach to 'Quantity' implicatures has some serious problems. The Gricean maxim derives only weak implicatures of the form 'the speaker doesn't know that φ'. The strengthening of these to implicatures of the form 'the speaker knows that not φ' involves an extra premise which in some cases amounts to being the strong implicature **itself**. Moreover, I show that the derivation mechanism wildly overgenerates non existent implicatures.

The Gricean theory has been modified, in particular in the work of Larry Horn, by restricting the application of Grice's first maxim of Quantity to contextually given ordering scales. I discuss a variety of problems for approaches like Horn's, in particular: the stipulative nature of the contents of the scales, various problems of overgeneration, and the problem of embedded implicatures.

I then develop an alternative to the Gricean theory of these implicatures. My starting point is the exhaustivity operator, *exh*, stipulated by Groenendijk and Stokhof within their theory of questions. I show that this operator already accounts for some of the 'Quantity' implicatures, without having the problems of the Gricean theories. *Exh* is supposed to have the meaning of *only*, but the semantics Groenendijk and Stokhof suggest for it is not sophisticated enough. I show some problems it runs into, and replace their semantics of *exh* with a new one that succeeds in the cases where theirs fails. I then show that my semantics for *exh* naturally generalizes to domains that Groenendijk and Stokhof did not cover.

The operation *exh* forms the heart of my alternative to the Gricean theory. I suggest to replace the Gricean theory of Quantity implicatures with a theory which consists of an exhaustivity operator, *exh*, that strengthens the meaning of a statement relative to a question, and a strong maxim of Quality for questions: "Answer the question!" I assume that answering a question means giving a (true and complete) semantic answer to it, and that *exh* is generally available, and can be used as a strategy to satisfy the strong Quality maxim for questions. This is how the effects of implicatures come in.

I show that whatever we need of the Horn scales comes out naturally from the semantics of the exhaustivity operator (it imposes a semilattice structure on the domain it operates on). Thus, the right kind of Horn scales, where needed, follow from the present theory.

I show that the resulting theory provides a satisfactory alternative account of scalar **and** clausal implicatures, and moreover, I show that the facts about implicature inheritance and cancellation in logically complex sentences ("the projection problem for implicatures") come out naturally within the framework suggested here.

Table of Contents

Chapter 1

Problems for Gricean Pragmatics

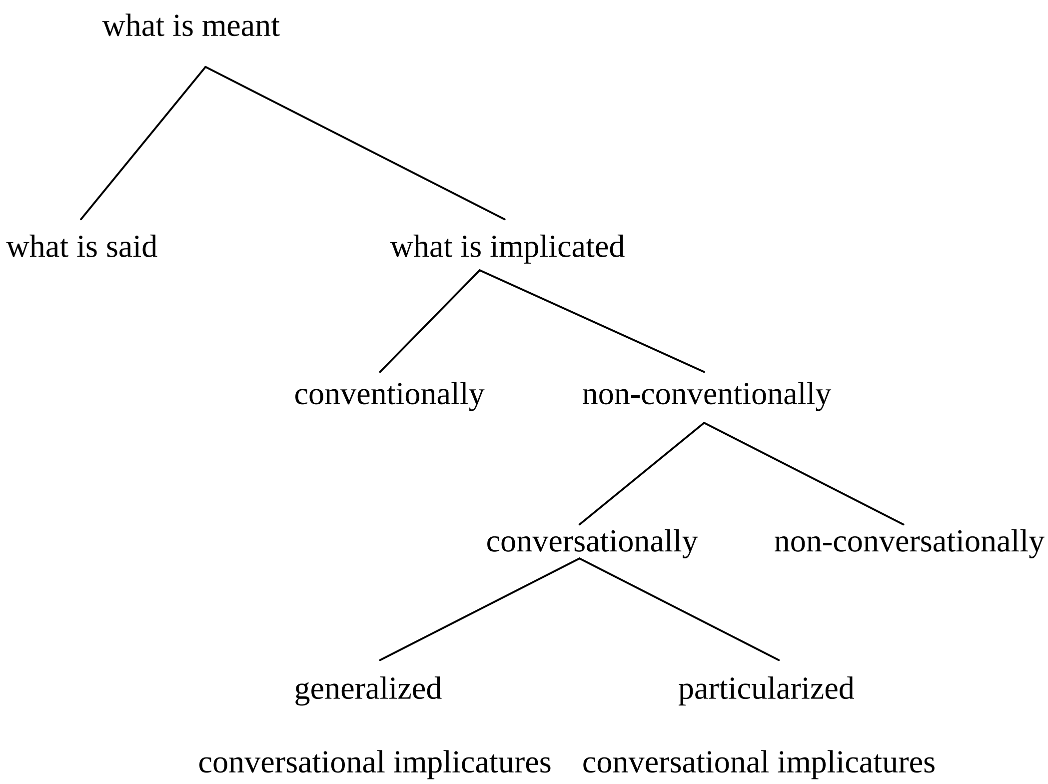
1.1 Conversational implicatures

The idea that meaning (in the pre-theoretical, non-technical use of the word) is not homogenous; i.e. that it can be divided into different 'layers', and that it involves the integration between different systems of rules, principles, constraints or otherwise, turned out to be very useful in the formulation and delineation of meaning theories, and it has a long history. Frege (1892) distinguishes the 'sense' of a sentence (the 'thought' expressed by it) from three other components of interpretation: presuppositions, varieties of interpretation which don't contribute to the sense of a sentence, but 'only illuminate it in a peculiar fashion' (such as the contrastive contribution of *although* and *but*, according to him), and 'subsidiary thoughts'. An example of the latter is the causal implication of (1).

(1) Napoleon, who recognized the danger to his right flank, himself led his guards against the enemy position.

According to Frege, sentence (1) expresses two thoughts, the thought that Napoleon recognized the danger to his right flanks, and the thought that Napoleon himself led his guards against the enemy position. The implication that the knowledge of the danger was the **reason** why Napoleon led the guards against the enemy, is probably not ‘really expressed’, but only ‘merely suggested’, because the sentence is probably true also in cases where there is no causal connection between the two parts of sentence (1).[1]

Grice (1975) suggests the following more elaborated distinction (adapted from Horn 1988):



It is not my intention to argue in favor or against this elaborate picture. I’m concerned with various types of inferences, which, since Grice, were put together under the rubric “conversational implicatures” (either generalized or particularized). Let me first introduce (at this stage, informally) three types of inference: implication, entailment and implicature. Following Chierchia and McConnell-Ginet (2000), and Kadmon

[1] Potts 2005 treats the contribution of nonrestrictive relative clauses like that in (1) as an instance of conventional implicature, although he does not discuss the implication of the relation between the proposition expressed by the relative clause and that expressed by the main clause.

(2001) I'll use the terms 'imply' and 'implication' very generally; "*A* **implies** *B*" means that *A* (or the utterance of *A*) gives some reason to conclude *B*. The terms 'entailment' and 'entail' will be reserved for a semantic relation; *A* **entails** *B* iff the information that *B* conveys is contained in the information that *A* conveys. I'll use the terms 'implicature' and 'implicate' for any implication which is not an entailment (more precise definitions and distinctions will be formulated later on). For example, while (2) implies both (3) and (4), it entails (3), and implicates (4).

(2) Not everyone danced.

(3) Someone didn't dance.

(4) Someone danced.

(2) does not entail (4), because (2) is compatible with a situation in which no one danced; nevertheless it strongly suggests (4).

It is not always easy to determine whether some implication is an entailment or an implicature. Consider (5):

(5) Maria and Alberto are married.

(6) Maria is married, and Alberto is married.

(7) Maria and Alberto are married to each other.

In many contexts we understand (5) as conveying (7), while in some contexts we understand it as conveying (6). One possibility is to assume that (5)'s meaning is equivalent to (6), and that it merely implicates (7). Another possibility is to assume that (5) is ambiguous between a collective and a distributive interpretations of *marry*. Under this assumption, (7) is an entailment of one of (5)'s readings (the reading where *marry* is a collective predicate), while (6) is entailed by both readings. Choosing between these two options is not a trivial matter. Adherents of the implicature theory would have to come up with a convincing and general method for deriving (7) as an implicature of (5), while adherents of the ambiguity theory would need to come up with convincing arguments for their view.

Grice (1975) suggests a framework for explaining how certain implicatures may be derived. He reserves the term 'conversational implicature' for a type of inference which is defeasible (can be cancelled without contradiction) and which can be derived from the interaction of the utterance's truth conditions (in its context of utterance), contextual assumptions, and principles that guide exchange of information in conversation. These principles are stated in his Cooperative Principle and its four maxims of conversation (Grice mentions the possibility of the existence of other maxims as well).

The Cooperative Principle:

Make your contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged. (Grice 1975: 45)

The Maxims of Quantity:

1. Make your contribution as informative as is required (for the current purposes of the exchange).

2. Do not make your contribution more informative than is required. (Grice 1975: 45)

The Maxim of Quality:

Try to make your contribution one that is true.

1. Do not say what you believe to be false.
2. Do not say that for which you lack evidence. (Grice 1975: 46)

The Maxim of Relation:

Be Relevant. (Grice 1975: 46)

The Maxim of Manner:

Be perspicuous.

1. Avoid obscurity of expression.
2. Avoid ambiguity.
3. Be brief (avoid unnecessary prolixity).
4. Be orderly (Grice 1975: 46)

Grice suggests that the derivation of conversational implicatures is as follows:

“A man who, by (in, when) saying (or making as if to say) that *p* has implicated that *q*, may be said to have conversationally implicated that *q*, PROVIDED THAT (1) he is to be presumed to be observing the conversational maxims, or at least the cooperative principle; (2) the supposition that he is aware that, or thinks that, *q* is required in order to make his saying or making as if to say *p* (or doing so in THOSE terms) consistent with this presumption; and (3) the speaker thinks (and would expect the hearer to think that the speaker thinks) that it is within the competence of the hearer to work out, or grasp intuitively, that the supposition mentioned in (2) is required.” (Grice 1975: 49-50)

Let me quote one of Grice’s examples to illustrate his point (the title and the comment in square brackets are mine). I chose this particular example, because it involves the first maxim of Quantity, which will be my main interest in this thesis. Implicatures that involve this maxim will be called ‘Quantity implicatures’.

<u>A holiday in France</u> (Grice 1975, 51-52)

A is planning with B an itinerary for a holiday in France. Both know that A wants to see his friend C, if to do so would not involve too great a prolongation of his journey:

A: Where does C live?

B: Somewhere in the south of France.

(Gloss: There is no reason to suppose that B is opting out [from the Cooperative Principle]; his answer is, as he well knows, less informative than is required to meet A’s needs. This infringement of the first maxim of Quantity can be explained only by the supposition that B is aware that to be more informative would be to say something that infringed the maxim of Quality, ‘Don’t say what you lack adequate evidence for’, so B implicates that he does not know in which town C lives.)

1.2 Gamut on Quantity implicatures

Gamut (1991) try to explicate the pattern of reasoning exemplified by the example just quoted in more detail. According to Gamut, a conversational implicature of a sentence *A* is a logical consequence of the conditions under which *A* can be correctly used. Something like the Gricean maxims play a role in determining these conditions.

A speaker S makes correct use of a sentence *A* in order to make a statement before a listener L just in case:

(i) S knows that *A* is true;

(ii) S knows that L does not know that *A* is true.

(iii) S knows that *A* is relevant to the subject of the conversation;

(iv) For all sentences *B* of which *A* is a logical consequence (and which are not equivalent to *A*), (i)-(iii) do not all hold with respect to *B*. (Gamut 1991, 205)

In (i)-(iii) above, Gamut actually use *believe*, not *know*. They point out explicitly that by *believe* they mean strict belief: "Not only must the speaker think it more probable that *A* holds than that *A* does not hold, he must also be quite convinced that *A* is indeed true" (Gamut, 205). As the meaning of the natural language verb *believe*, in my opinion, expresses a much weaker notion, I will not use it in this context. I chose *know* instead, for the following reason. I gather that the sentences *A* which Gamut consider here are not about the information or the beliefs of the speaker (for example, they are not of the form *I know that*… or *I believe that*…). If *A* were such a sentence, the clauses (i) - (iii) above would have to change to accommodate for this fact. When

S utters a sentence *A* which is not explicitly about her beliefs, L will normally assume that *A* follows from S's information. A response such as *How do you know that A?* to the utterance of *A*, is more natural than a response such as *Why do you believe that A?* The notion needed here is something like: "The context and conversation so far have put S in a position that she can be reasonably regarded as having to accept that *A* is true". Since this is by far too complex a notion to repeat more than once, and since I believe that the natural language verb *know* is sometimes (mis)used to express such a notion, I will (mis)use the word *know* to stand for this relation. Thus where I use *know*, it is without entailing that *A* is actually true.

We see that Gamut interpret informativeness in terms of asymmetrical entailment (*B* entails *A*, and it is not equivalent to *A*). This characterization needs some refinements, and this point will become clear in the example discussed below, but I leave a detailed discussion for a later stage. Let me reanalyze Grice's 'holiday' example in Gamut's terms.

(8) C lives somewhere in the south of France

A= C lives somewhere in the south of France.

B = C lives in Nice.

For simplicity let us assume that *B* entails *A* (*B* does not strictly entail *A*, *A* is entailed by *B* plus the information that Nice is in the south of France. Let us assume that both S and L have this information, and that they know that the other has it as well).

S makes a correct use of ‘C lives somewhere in the south of France’ before L just in case:

(i) S knows that C lives in the south of France;

(ii) S knows that the information that C lives in the south of France would be new to L;

(iii) S knows that the fact that C lives in the south of France would be relevant to the subject of the conversation;

(iv) Either: (a) S does not know that C lives in Nice; or

(b) S does not know that the information that C lives in Nice would be new to L; or

(c) S does not know that the fact that C lives in Nice would be relevant to the subject of the conversation.

Concerning the possibilities in (iv), we can rule out (c), because the information that C lives in Nice would be relevant here. (b) must also be ruled out; if S knows that the information that C lives in the south of France is new to L (premise ii), so must be the information that C lives in Nice (this is true under the assumption that both S and L know that Nice is in the south of France, and that S knows that L knows this). So, we must conclude that (a) is true; i.e. S doesn’t know that C lives in Nice. This line of argumentation works for any town, X, hence for every X, S does not know that C lives in X; i.e. S does not know in which town C lives.

Note also that in contexts where (c) is true (S does not believe/know that the fact that C lives in Nice would be relevant to the subject of the conversation), the theory `predicts that there is no implicature that S doesn't know in which town C lives. This seems right:

(9) L is planning a high school reunion, but he'll invite only those old class mates who are not living presently abroad.

L: Where does C live?

S: Somewhere in the South of France.

In (9), S's utterance does not implicate that S doesn't know in which town C lives. In this case the information that C lives abroad (which is entailed from S's utterance in the context given) is already sufficient for the purpose of L.

1.3 Problems with Gamut's theory

1.3.1 Strong vs. weak Quantity implicatures

Let us check whether Gamut's proposal can derive (11) as a conversational implicature of (10). That is, we assume that (10) actually intuitively does implicate (11), and we check whether Gamut predict this.

(10) Mary has a cat or a dog.

(11) Mary doesn't have a cat **and** a dog.

It is easy to see that an implicature that Gamut can derive for (10) is (12):

(12) S doesn’t know that Mary has a cat and a dog.

Obviously, this is too weak. Note, however, that (11) does follow from (12’) below, under the assumption that S’s information is true.

(12’) S knows that Mary doesn’t have a cat and a dog

It is widely assumed in the literature about Quantity implicatures (see, for example, Gazdar 1979 and Levinson 1983) that the Quantity implicatures of a proposition ψ, are epistemically modified, and it is observed that Grice’s maxim of Quantity sometimes licences strong inferences of the form ‘S knows that not ϕ’, and sometimes only weak inferences of the form ‘S doesn’t know that ϕ’, where ϕ is a stronger alternative to ψ. In this, the literature follows the discussion of this phenomenon in Horn (1972). Levinson (1983) states that this fact remains one of the many mysteries in the area. Note that Gamut derives only weak inferences such as (12), but if we could motivate a contextual strengthening of (12) to (12’), we would have a plausible way to explain the inference from (10) to (12’) and to (11) as a conversational implicature. (12), of course does not entail (12’), because (12’) is also true if (13) is true.

(13) S knows that Mary has a cat or a dog, but doesn’t know whether she has both.

In order to rule out this option, we must add the following additional premise:

(14) S knows whether Mary has a cat and a dog.

Is this additional premise plausible? This premise means one of the following two:

i) S knows that Mary has a cat and a dog

ii) S knows that Mary doesn't have a cat and a dog.

But case (i) contradicts (12), the weak implicature derived by Gamut. If (i) were true, S should have said: *Mary has a cat and a dog*, and not the weaker statement (10). Thus, case (i) is ruled out, because uttering (10) in a context where (i) is true, is not correct according to Gamut. This means that the additional premise reduces to (ii). But this, of course, **is** the strong implicature (12'). This means that Gamut can only derive the strong implicature in contexts where the strong implicature is already assumed. Hence, it means that Gamut **cannot** derive the strong implicature.

1.3.2 The need to constrain the derivation process

A closer look at Gamut's Gricean theory shows that even if the problem of deriving strong implicatures is solved somehow, the theory still has a very serious problem. It would derive non-existent and even contradictory implicatures. In the same way that the theory is meant to predict that (10) conversationally implicates (11), it would also make the wrong (and incompatible) prediction that it implicates (15).

(15) S knows that Mary has a cat and a dog.

Let us see why.

Since *(A and B)* asymmetrically entails *(A or B)*, we get the implicature: *S doesn't know that (A and B)*, and this should somehow strengthen to *S knows that not (A and B)*. But the same procedure should apply to other entailments as well. Consider exclusive *or*. Clearly, *(A or B) and [not (A and B)]* entails *(A or B)*. So Gamut get the implicature (a): *S doesn't know that {(A or B) and [not (A and B)]}*. This should strengthen to (b): *S knows that not {(A or B) and [not (A and B)]}* which is equivalent to (c): *S knows that {[not(A or B)] or (A and B)}*. But by Quality, condition (i) in Gamut's formulation, it holds that (d): *S knows that (A or B)*. From (c) and (d) it follows that *S knows that A and B*. This is obviously wrong. The utterance of *(A or B)* doesn't have the implicature *S knows that A and B.*

Assuming there is a general strengthening procedure which applies to all weak implicatures, Gamut's theory derives too many. A way to constrain the derivation process is needed, otherwise it is completely unclear when we can and when we cannot apply the strengthening procedure.

1.4 Constraining informativeness by Horn-scales

A solution to the problem discussed above may be found in Horn's (1972) notion of a scalar implicature. Horn focuses on the type of implicatures that arise with sentences that contain some value on a quantitative scale. Horn modifies the Gricean theory by introducing the concepts of a pragmatic scale and a scalar implicature. Horn defines the notion of a *scalar implicature* as follows: when a speaker utters a statement S which contains an element p_i on a given quantitative scale $p_1, p_2, \ldots, p_n$ (which orders the elements $p_1, p_2, \ldots, p_n$ according to a relation of ‘is more informative than’), the hearer **can** infer that every statement S', which is the result of substituting p_i in S for a higher value in the same scale, is false, and she **must** infer that a statement S'', which is the result of substituting p_i in S for the highest value in the same scale, is false.

Let us apply Horn’s definition to example (10), which is repeated below.

(10) Mary has a cat or a dog.

Suppose that (10) is asserted, and suppose, with Horn, that *or* is an element on the following scale: *or, and*. Then, the hearer must infer (16).

(16) It is not the case that Mary has a cat and a dog.

(17) It is not the case that Mary has a cat or a dog, but not both.

In order to block the inference in (17), we have to assume that while the pair <α or β, α and β> is a ‘legitimate’ quantitative scale, the pair <α or β, (α or β) and not (α and β)> is not.

If the application of Grice’s first Quantity maxim can be restricted to ‘legitimate’ quantitative scales, we have solved our problem. But we are faced with a new one, namely, what forms a ‘legitimate’ quantitative scale? Can we define Horn scales?

We see that, contrary to Gamut (1991), who derive only weak Quantity implicatures, Horn (1972) derives only the strong ones. I would like to make it clear at this stage that I do not intend to replace Gamut's theory with Horn's. Gamut's derivation process preserves Grice's original insight that Quantity implicatures do not arise independently of 'the purpose of the conversation'. Contrary to Horn, Gamut is capable of handling cases when the context doesn't give rise to a scalar implicature. My intention is to check whether there is a way to modify Gamut's theory with Horn's scale insight in a way that will appropriately constrain it.

Horn himself is aware of the role of context in building scales. See for example (18) and (19).

(18) Arnie has 3 children, if not 4.

(19) Arnie is capable of breaking 70 on this course, if not 65.

Having 4 children entails having 3, while in a golf game, achieving a lower score entails that it is possible to achieve a higher one, so the scale of numbers is reversed.

Here is a nice example that Horn (1989, p.241) cites from a review of *The Soong Dynasty* by Sterling Seagrave: "The picture of Chang Kai-Shek that emerges is one that rivals Mussolini, if not Hitler, as the very model of a modern dictator." Horn comments that while there are no semantic criteria for putting proper names on a scale, this example suggests that there is one on which dictators are ranked, and that "the Führer clearly outranks (outgrosses?) il Duce on this scale".

Quantitative scales may be based on a simple entailment relation (as in the case of numbers, where usually the higher number on the scale entails the lower number), or on more refined notions. If we want to explain the inference from (20) to (21) and from (22) to (23) (in the context of Grice's original example) on the grounds that the speaker couldn't be as informative as expected, we must come up with a more contextual notion of informativeness.

(20) Some of my friends are Zoroastrians.

(21) S knows that not all her friends are Zoroastrians.

(22) C lives somewhere in the south of France.

(23) S doesn't know that C lives in Nice.

How should we characterize the relation between *some* and *all* and between *the south of France* and *Nice*? Let me get more precise. For the terms 'entailment' and 'entails' I'll use the standard definition from modal logic.

(24) A sentence *A* (in a language *L*) entails a sentence *B* iff for every model *M* for *L*, for every world *w* in *M*, if *A* is true in *w*, then *B* is true in *w* (for every M and w, $[\![A]\!]_{M,w} \subseteq [\![B]\!]_{M,w}$).

It is easy to see that the sentence *C lives somewhere in the south of France* does not entail the sentence *C lives in Nice* because there are worlds where Nice is not in the south of France. But, as mentioned before, the inference does hold if we restrict ourselves to worlds, like the real world, where Nice is in the south of France. In order to define this context dependent notion of entailment, I'll use Stalnaker's (1974, 1975, 1978) notion of a 'context set'. Stalnaker represents the common beliefs and assumptions of the participants in the conversation about the world as a set of possible worlds. This set contains the worlds compatible with what is assumed in the conversation. Each world in this set, which is called the context set, could be, as far as the speakers assume, the real world. The notion of entailment relevant to the first maxim of Quantity is entailment in a context set.

(25) Sentence *A* (in a language *L*)entails sentence *B* in a speech context *c*, iff for every model *M*, for every world *w* in the context set *C* of *c*, if *A* is true in *w*, then *B* is true in *w* (for every M and w, $[\![A]\!]_{M,w} \cap C \subseteq [\![B]\!]_{M,w}$).

Now we turn to the relation between *All my friends are Zoroastrians* and *Some of my friends are Zoroastrians*. The first sentence is compatible with S not having friends at all, the second is not, and hence the former does not entail the latter. However, the inference does go through in every context where the set of S's friends is not empty. Moreover, using the first sentence in a context where it is known that S doesn't have friends is bizarre. In normal cases, the context set will include only those worlds in which the set of S's friends is not empty, or otherwise this information will be accommodated (in the sense of Lewis 1979).

The two cases discussed are similar in that they are both instances of entailment in a context set. They differ as follows: It is quite normal for S to assert *C lives in Nice* even if she doesn't know that Nice is in the south of France or expects L to know it.

However S would not normally assert *All my friends are Zoroastrians* if she knows she hasn't got any friends, and she would normally expect L to accommodate this information, if she thinks L doesn't assume it already.

What we just did is give a contextual notion of inference which is more suitable than strict entailment for characterizing what it means for a sentence to be more informative than another. However, the problem of constraining the applicability of this notion, so that it won't give rise to non-existing and contradictory implicatures, still remains. What should be the restriction on Horn-scales, so, that, for example, <α or β, α and β> will be a legitimate scale but not <α or β, (α or β) and not (α and β)>?

There are many attempts in the literature to constrain the notion of a Horn-scale. While Gazdar (1979, p.58) assumes that "the scales are, in some sense, 'given to us'", Hirschberg (1985) proposes to substitute 'scale' with any partial ordering relation which is mutually believed salient by the speaker and the listener. The problem is that I don't see why the relation between *or* and *and* should be always more salient than the relation between *or* and *or but not and* (i.e. exclusive disjunction).

Atlas and Levinson (1981) require that items on the scale should be from the same semantic field, lexicalized to the same degree, and have the same brevity. Matsumoto (1995) shows that this is wrong. First, items on a Horn scale need not be from the same semantic filed.

(26) A: What have you done with that mail?

B: I've typed it

Implicature: B has not mailed it yet.

Matsumoto argues that it is not clear that a set of words like *type* and *mail* can be regarded as forming a semantic field. He points out that members of the same semantic field are often all relevant to the discourse in which one of them is used, but the real condition here is of relevance and not of forming a semantic field.

Second, there is no brevity/lexicalization condition on Horn scales. As (27) shows, an implicature on the basis of the scale <warm, a little bit more than warm> is possible.

(27) It was warm yesterday, and it is a little bit more than warm today.
Implicature: The speaker believes that it was not “a little bit more than warm” yesterday.

Another case that shows that brevity will not constrain Horn scales properly is the following: *Mary*, as an answer to the question *Who is writing a PhD in linguistics?*, implicates (if a full answer is expected), that John is not writing a PhD in linguistics, and it can’t implicate that he is. This suggests that while <Mary, Mary and John> can function as Horn-scale in the right context, <Mary, only Mary> can’t. But obviously, *only Mary* is shorter than *Mary and John*.

Matsumoto argues that a solution may be found in Horn’s (1989) claim that ‘positively scalar’ and ‘negatively scalar’ elements cannot be put on the same scale. Horn excludes the possibility of *some* and *not all* or *possible* and *unlikely* to be on the same scale. According to Horn, the notions of ‘positively scalar’ and ‘negatively scalar' are parallel to the semantic notions of upward and downward monotonicity, which play a crucial rule in the distribution of negative polarity items (see Ladusaw 1979). Matsumoto interprets Horn's condition as requiring that all elements in a Horn-scale should be either upward monotone or downward monotone.

Before giving the formal definitions of upward and downward monotonicity for the general case, let me explain these notions for a simple case.

(28) Let O be a function from propositions to propositions,

O is upward monotone iff for every two propositions φ, ψ such that φ entails ψ, $O(\varphi)$ entails $O(\psi)$.

O is downward monotone iff for every two propositions φ, ψ such that φ entails ψ, $O(\psi)$ entails $O(\varphi)$.

It is easy to see that *it is possible that* is upward monotone, while *it is unlikely that* is downward monotone. Given that *Mary is running* entails *Mary is moving, it is possible that Mary is running* entails *it is possible that Mary is moving*, and *it is unlikely that Mary is moving* entails *it is unlikely that Mary is running*. If we want to generalize this notion to other kinds of functions, we must first define entailment also among entities other than propositions.

In defining entailment and monotonicity, I use a logical language used in Kratzer (1991) and Kadmon (2001) which is based on Cresswell (1973). The syntax of the language is very similar to extensional type logic, the crucial semantic difference being that expressions of type t, i.e., formulas, do not denote truth values, but sets of possible worlds. Note that in this language the type of one place predicates, <e,t>, is a function from individuals to sets of possible worlds.

I chose this language, because it is easier to define and demonstrate the notion of entailment between expressions other than propositions using it. This notion, of course, is translatable to more standard languages such as intensional type logic (IL) and two sorted type logic (TY2) which I'll use in the next chapters of this dissertation. Definitions of generalized entailment can be found in Ladusaw (1979) and Groenendijk and Stokhof (1989) (without entailment defined at type e).

(29) Let e be the type of individuals, and t be the type of propositions (sets of possible worlds).

(i) For denotations of type e, entailment is the part-of relation between plural individuals or mass entities.

(ii) For denotations of type t, entailment is the subset relation.

(iii) If α and β are denotations of any type <a,b>, such that entailment is defined between denotations of type b, then α entails β iff for every denotation, γ, of type a, $\alpha(\gamma)$ entails $\beta(\gamma)$.

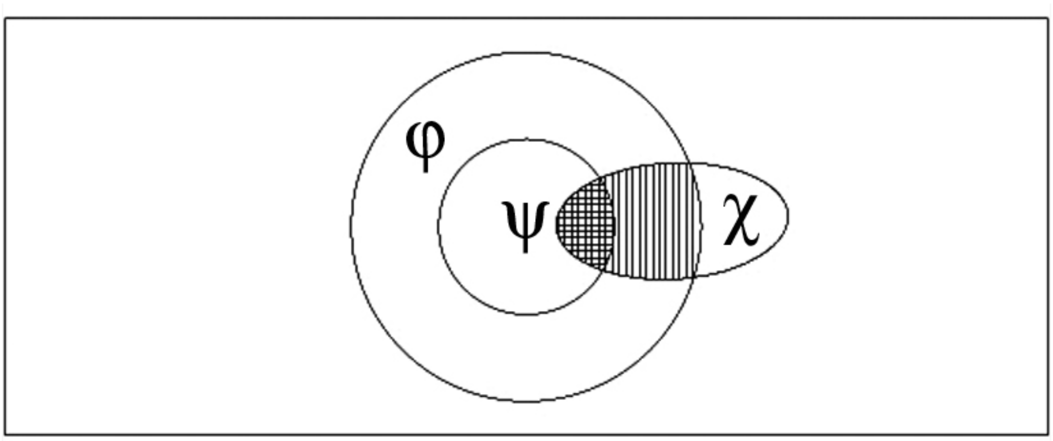
(30) Let f be a function of type <a,b>:

f is upward monotone iff for every α, $\beta \in a$, if α entails β, then $f(\alpha)$ entails $f(\beta)$

f is downward monotone iff for every α, $\beta \in a$, if α entails β, then $f(\beta)$ entails $f(\alpha)$

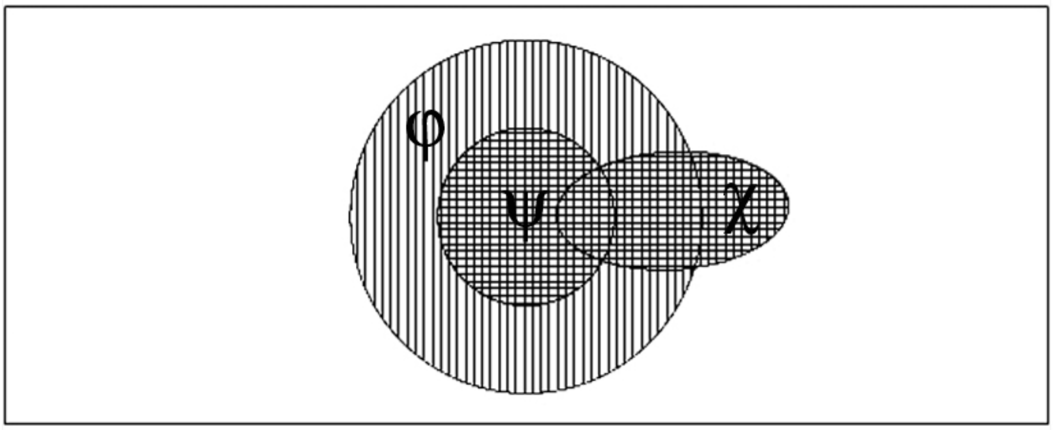
I’ll show that the functions from type <t, <t,t>> that are the denotations of *and* and *or* are upward monotone, and that the function from type <t,<t,t>> expressible by *or but not and* is not upward monotone. *And* is the intersection relation between sets of possible worlds, *or* is the union relation.

Let φ, ψ be propositions such that φ entails ψ. We need to check whether andφ entails andψ. According to (29iii), this amounts to checking whether and$\varphi(\chi)$ entails and$\psi(\chi)$ For every proposition χ. This is indeed the case, as shown in diagram A.
The intersection between ψ and χ is a subset of the intersection between φ and χ.



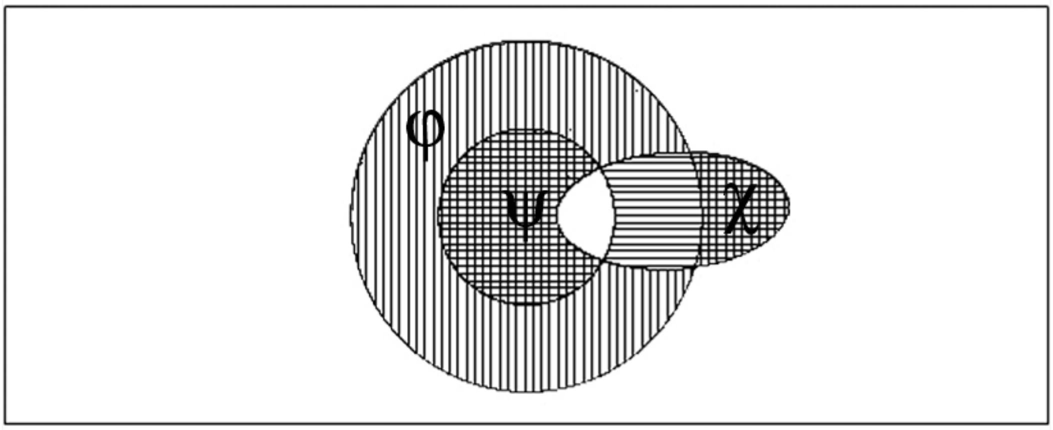
A

As shown in diagram B, *or* is also upward monotone. The union of ψ and χ is a subset of the union of φ and χ.



B

Exclusive *or* is not upward monotone, as shown in diagram C, the union of ψ and χ minus their intersection, is not a subset of the union of φ and χ minus their intersection.



C

I'll show that the functions from type <<e,t>, t> denoted by *John* and *John and Mary* are both upward monotone, whereas the function from type <<e,t>,t> denoted by *only John* is not.

⟦*John*⟧ = λP.P(j)

⟦*John and Mary*⟧ = λP.P(j) ∧ P(m)

$[\![$*only John*$]\!] = \lambda P.P(j) \wedge \neg\exists x[x{\neq}j \wedge P(x)]$

Let P and Q be two expressions from type <e,t> such that P entails Q. We need to check whether $[\![$*John*$]\!]$(P) entails $[\![$*John*$]\!]$(Q); i.e. that P(j) entails Q(j). This is indeed so, because P entails Q, and according to definition (29iii) that means that for every individual k, P(k) entails Q(k).

Let P and Q be two expressions from type <e,t> such that P entails Q. We need to check whether $[\![$*John and Mary*$]\!]$(P) entails $[\![$*John and Mary*$]\!]$(Q); i.e. that $P(j) \wedge P(m)$ entails $Q(j) \wedge Q(m)$. This is indeed so, because P(k) entails Q(k), for every k, hence the intersection of P(j) and P(m) is a subset of the intersection of Q(j) and Q(m).

Let P and Q be two expressions from type <e,t> such that P entails Q. We need to check whether $[\![$*only John*$]\!]$(P) entails $[\![$*only John*$]\!]$(Q). This is not the case. Let P be *run* and let Q be *move*. If only John runs, he needn't be the only one who moves.

I will show that the function denoted by *some* is upward monotone, and that the functions denoted by *all* is downward monotone. These functions are of type <<e,t>, <<e,t>, t>>.

Let P and R be two expressions from type <e,t>, such that P entails R. We need to check whether $[\![$*some*$]\!]$(P) entails $[\![$*some*$]\!]$(R). By definition (29iii), we need to check whether $[\![$*some*(P)$]\!]$(Q) entails $[\![$*some*(R)$]\!]$(Q) for every expression Q from type <e,t>. This is indeed so. In the set of worlds where some P is Q, that P is also R, because all P's are R's.

Let P and R be two expressions from type <e,t> such that P entails R. We need to check whether ⟦all⟧(R) entails ⟦all⟧(P). By definition (29iii), we need to check whether ⟦all(R)⟧(Q) entails ⟦all(P)⟧(Q) for every expression Q from type <e,t>. This is indeed so. In the set of worlds where all R's are Q's all P's are Q's as well because all P's are R's.

I'll leave it to the reader to check that the functions from type <<e,t>, t> denoted by *Some semanticists* and *All semanticists* are both upward monotone, whereas the function from type <<e,t>,t> denoted by *No semanticists* is downward monotone.

We must assume that the functions of type <e,t> denoted by *run* and *run quickly* are both upward monotone. Here's why. Let δ and ε two individuals such that $\varepsilon \sqsubseteq \delta$ (for example, δ= John and Mary and ε = Mary). Our intuitions about entailment suggest that for any δ entailing ε, if ⟦run⟧(δ)= 1, then ⟦run⟧(ε)= 1. The same goes for *run quickly*. This follows from our intuitions that *run* and *run quickly* are **distributive** predicates.

Now we are in a position to define a Horn-scale, and see how this definition constrains the derivation of Quantity implicatures.

(31) Let p_1, p_2, …p_n be denotations of the same type. <p_1, p_2,…p_n> is a Horn-scale in a speech context c, if the following two conditions hold:

i. For every i, i>0, p_{i+1} entails p_i in c, and p_i does not entail p_{i+1} in c.

ii. p_1,p_2,…p_n are all upward monotone or p_1,p_2,…p_n are all downward monotone.

Concerning the cases we checked before, <⟦some⟧, ⟦all⟧>, <⟦or⟧, ⟦or but not and⟧> and <⟦John⟧, ⟦only John⟧> are not Horn-scales in any context, whereas <⟦or⟧, ⟦and⟧>, <⟦run⟧, ⟦run quickly⟧> and <⟦John⟧, ⟦John and Mary⟧> are Horn-scales in every context, and <⟦some semanticists⟧, ⟦all semanticists⟧> is a Horn-scale in a context where there are semanticists.

We already saw that implicatures based on *or*, *and* and on *John*, *John and Mary* are possible, while implicatures based on *or*, *or but not and* and on *John*, *only John* are impossible. We also saw implicatures based on *all* and *some*, and they can be explained now on the basis that <⟦all X's⟧, ⟦some X's⟧> is a Horn-scale in a context where *X* is not empty. Do we get implicatures based on predicted scales like <⟦run⟧, ⟦run quickly⟧> or <⟦run⟧, ⟦run slowly⟧>? Out of the blue, sentence (32) does not implicate (33).

(32) Mary ran.

(33) Mary didn't run quickly/slowly.

However, it seems that in a context were we contrast the runners with the quick or slow runners, such implicatures do exist.

(34) Mary, John and Bill ran. John ran quickly, Bill ran slowly.

L can infer that S knows that Mary didn't run quickly, and that she didn't run slowly, or at least that S doesn't know that she ran quickly, and that S doesn't know that she ran slowly. If we are to explain these inferences as Quantity implicatures, we need to come up with an explanation why the scales <⟦run⟧, ⟦run quickly⟧> or <⟦run⟧, ⟦run slowly⟧> are invoked only in few contexts, while the scales <⟦or⟧, ⟦and⟧> or <⟦some semanticists⟧, ⟦all semanticists⟧> are invoked out of the blue.

Reformulating Gamut's Gricean theory, so it would reflect Horn's (1989) and Matsumoto's (1995) monotonicity requirement, means, informally, that the process should consider not all sentences that are entailed by and not equivalent to S's utterance (as in Gamut's original formulation), but only those which denote a proposition that is obtained from S's original utterance by substituting some element for a higher element on the same Horn-scale. Thus, in deriving Quantity implicatures we cannot just refer to propositions, as in Gamut's original theory, but need to access the parts of the proposition and its semantic composition. Here's an attempt to give a more precise formulation of the process.

(35) **The Gamut-Horn-Matsumoto Theory**:

1. A conversational implicature of a sentence *A* is a logical consequence of the conditions under which *A* can be correctly used.

2. A speaker S makes correct use of a sentence $A=(\alpha_1(\ldots(\alpha_{i-1}(\alpha_i(\alpha_{i+1}(\ldots(\alpha_n)\ldots)$ in order to make a statement before a listener L just in case:

i. S knows that *A* is true;

ii. S knows that L does not know that *A* is true.

iii. S knows that *A* is relevant to the subject of the conversation;

iv. For all propositions *B* which differ from the proposition denoted by A as follows: B=($[\![\alpha_1]\!]$ (…($[\![\alpha_{i-1}]\!]$ ($[\![\beta]\!]$ ($[\![\alpha_{i+1}]\!]$ (…($[\![\alpha_n]\!]$)…), where $< [\![\alpha_i]\!], [\![\beta]\!] >$ is a Horn scale in the context c, (i)-(iii) do not all hold with respect to *B*.

Before discussing some problems with this, I'll demonstrate how it works in two cases.

1.5 Two detailed examples: Quantity implicatures in conditionals and disjunctions

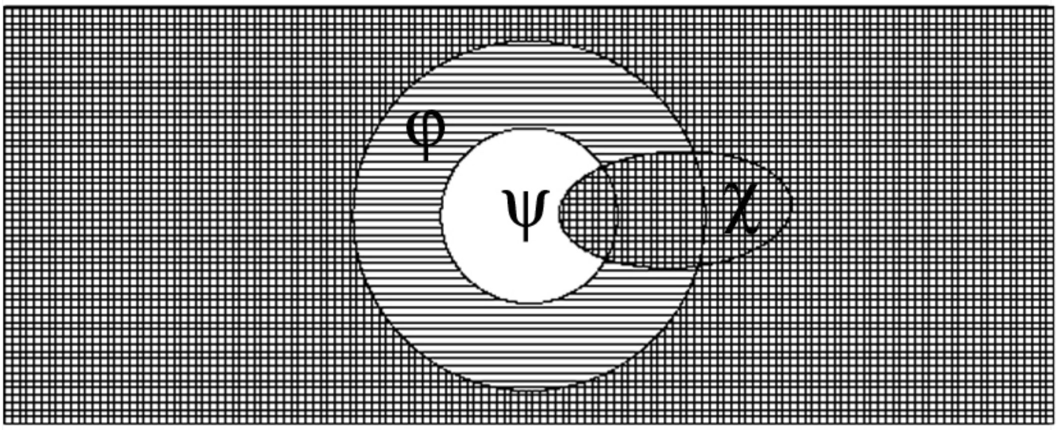
I will show now what implicatures the Gamut-Horn-Matsumoto process predicts for conditionals and disjunctions. For the sake of the illustration, I assume that the alternatives which the process considers are built only on the basis of the functions denoted by *if…then* and *or*.

1.5.1 Conditionals

For simplicity, let's assume that the natural language conditional operator denotes the material implication, a function of type <t, <t,t>> with the following meaning:

(36) $\lambda\varphi\lambda\psi(\varphi\rightarrow\psi)$; $[\![\varphi\rightarrow\psi]\!] = (W- [\![\varphi]\!])\cup [\![\psi]\!]$

This function is downward monotone. Here's why. Let φ, ψ be propositions such that φ entails ψ. We need to check whether $\to\psi$ entails $\to\varphi$. According to (29iii), this amounts to checking whether $\to\psi(\chi)$ entails $\to\varphi(\chi)$, for every proposition χ. This is indeed the case, as shown in diagram D. The union of χ and the complement of φ is a subset of the union of χ and the complement of ψ.



D

It is not very hard to see that out of the other fifteen possible denotations of type <t,<t,t>>, only the following seven entail the material implication $\lambda\varphi\lambda\psi(\varphi\to\psi)$:

(37) a. $\lambda\varphi\lambda\psi(\varphi\wedge\neg\varphi)$

b. $\lambda\varphi\lambda\psi(\varphi\vee\psi)$

c. $\lambda\varphi\lambda\psi(\psi)$

d. $\lambda\varphi\lambda\psi(\varphi\leftrightarrow\psi)$

e. $\lambda\varphi\lambda\psi(\neg\varphi)$

f. $\lambda\varphi\lambda\psi(\neg\varphi)\wedge(\neg\psi)$

g. $\lambda\varphi\lambda\psi(\psi\to\varphi)$

Out of these seven functions, only five are downward monotone: functions a, c, e, f and g. So, the only propositions which are stronger than the conditional *if φ, then ψ* which our process should consider are:

(38) a. The proposition expressed by any contradiction.

b. ψ

c. $\neg\varphi$

d. $(\neg\varphi)\wedge(\neg\psi)$

e. $\psi\rightarrow\varphi$

The contradiction is not informative in any context, hence I think we can ignore it. Let me start with the implicatures based on (38b) and (38c). Assuming a context in which *S* thinks that (38b) and (38c), are relevant and new to *L*, *L* would get the following implicatures of *if φ, then ψ*:

(39) S doesn't know that ψ.

(40) S doesn't know that not φ.

Assuming there exists some general process of strengthening weak implicatures of the form *S doesn't know that p* to strong implicatures of the form *S knows that not p*, we might expect (39) and (40) above to be strengthened to (41) and (42):

(41) S knows that not ψ.

(42) S knows that φ.

The strong implicatures are not derived in this case. Here's why. From (41) and *S*'s original utterance, *if φ, then ψ*, *L* would have to infer that *S* knows that not φ. But this would contradict (40). Similarly, from (42) and the utterance of *if φ, then ψ*, *L* would have to infer that *S* knows that ψ. But this would contradict (39).

Thus the Gamut-Horn-Matsumoto theory predicts the clausal implicatures of the conditional. (39) and (40), together with the fact that (41) and (42) are false mean that for all *S* knows it is possible that φ and it is possible that *not* φ, and that it is possible that ψ, and it is possible that *not* ψ.

Now we turn to (38d) and (38e). The weak implicatures that can be derived on the basis of these propositions are (43) and (44) which follow already from (40) and (39) respectively, but it may be the case that (43) and (44) are strengthened to (45) and (46). Let us check this possibility.

(43) S doesn't know that not φ and not ψ

(44) S doesn't know that if ψ then φ

(45) S knows that φ or ψ

(46) S knows that if not φ then not ψ

(43) cannot be strengthened to (45). Here's why. From (45) and S's original utterance, *if φ, then ψ*, *L* would have to infer that *S* knows that ψ. But this would contradict (39). On the other hand, (45) can be strengthened to (46). This is called conditional perfection. From (46) and the utterance of *if φ, then ψ*, *L* will infer that *S*

knows that ψ *iff* φ. This inference does not contradict any of the weak implicatures, and it appears in many contexts, for example:

(47) If you study, you'll pass the exam

(48) Imp: If you don't study, you won't pass the exam.

Stipulating monotonicity, prevents *L* from inferring, for example, that *S* know that it is not true that φ *iff* ψ, because the function that the material equivalence denotes, function (37d) above, although entailing the material implication, is not downward monotone. This prediction is good. The utterance of a conditional *if* φ *then* ψ does not normally implicate ψ (whether φ or *not* φ).

This example was given mainly for the purpose of demonstration. I don't assume that the natural language conditional is actually the material implication. If conditionals are strict implications, $\Box(\varphi\rightarrow\psi)$, the picture will essentially stay the same, but we will have to make certain decisions about the relation between the necessity operator $\Box$, and the epistemic operator that we used in the implicature derivation process.

1.5.2 Disjunctions

What are our predictions concerning disjunctions? We saw earlier that the function denoted by sentential *or* is upward monotone. There are seven functions of type <t,<t,t>, that entail, but are not equivalent to, the denotation of *or*. Five of them are upward monotone:

(49) a. $\lambda\varphi\lambda\psi(\varphi\wedge\neg\varphi)$

b. $\lambda\varphi\lambda\psi(\varphi)$

c. $\lambda\varphi\lambda\psi(\psi)$

d. $\lambda\varphi\lambda\psi(\varphi\wedge\psi)$

e. $\lambda\varphi\lambda\psi(\varphi)\wedge(\neg\psi)$

So (ignoring the contradiction), the only propositions which are stronger than the disjunction *φ or ψ* which Gamut's process should consider are:

(50) a. φ

b. ψ

c. $\varphi\wedge\psi$

d. $(\varphi\wedge(\neg\psi))$

Let me start with (50a) and (50b). Assuming a context in which *S* thinks that (50a) and (50b) are relevant and new to *L*, *L* would get the following implicatures of *φ or ψ*:

(51) S doesn't know that φ.

(52) S doesn't know that ψ.

Assuming there exists some general process of strengthening weak implicatures of the form *S doesn't know that p* to strong implicatures of the form *S knows that not p*, we might expect (51) and (52) above to be strengthened to (53) and (54):

(53) S knows that not φ.

(54) S knows that not ψ.

The strong implicatures are not derived in this case. Here's why. From (53) and S's original utterance, *φ or ψ*, *L* would have to infer that *S* knows that *ψ*. But this would contradict (52). Similarly, from (54) and the utterance of *φ or ψ*, *L* would have to infer that *S* knows that *φ*. But this would contradict (51).

Thus the Gamut-Horn-Matsumoto theory predicts also the clausal implicatures of the disjunction. (51) and (52), together with the fact that (53) and (54) are false mean that for all *S* knows it is possible that *φ* and it is possible that *not φ*, and that it is possible that *ψ*, and it is possible that *not ψ*.

Now we turn to (50c) and (50d). The weak implicatures that can be derived on the basis of these propositions are (55), which already follows from both (51) and (52), and (56) which already follows from (51), but it may be the case that (55) and (56) are strengthened to (57) and (58) respectively. Let us check this possibility.

(55) S doesn't know that φ and ψ

(56) S doesn't know that φ and not ψ

(57) S knows that not (φ and ψ)

(58) S knows that not φ or ψ

(56) cannot be strengthened to (58). Here's why. From (58) and *S*'s original utterance, φ *or* ψ, *L* would have to infer that *S* knows that ψ. But this would contradict (52). On the other hand, (55) can be strengthened to (57). From (57) and the utterance of φ *or* ψ, *L* will infer that *S* knows that either φ *or* ψ but not both. This inference does not contradict any of the weak implicatures.

Stipulating monotonicity, prevents L from inferring, for example, that *S* know that it is not true that φ *or* ψ *but not both*, because the function that the exclusive disjunction denotes, as we saw earlier, although entailing the inclusive disjunction, is not upward monotone.

So far then, we have seen two cases where the Gamut-Horn-Matsumoto theory works quite well.

1.6 Problems with the Gamut-Horn-Matsumoto theory

As mentioned earlier, introducing monotonicity into the definition of Horn scales means that, in the derivation process of a Quantity implicature of a proposition *A*, one must have access to its parts and their semantic composition (this follows from the definition of monotonicity). One problem immediately presents itself: How do we know which part or parts of the sentence we should use in the derivation of implicatures? When we derive the Quantity implicatures of a certain proposition *A*, do we need to check all possible Horn scales that can be built on the basis of all the constituents of *A* or do we need to restrict ourselves to only certain constituents?

Let us consider (59) and (60):

(59) Three linguists are smart.

(60) It is not the case that four linguists are smart.

We can derive (60) as an implicature of (59) on the basis of the scale <three, four>, or, alternatively, on the basis of <three linguists, four linguists>. In addition, there is nothing to prevent us from using the scale <three linguists, three semanticists>, to derive (61) as an implicature of (59):

(61) It is not the case that three semanticists are smart.

But that is problematic. Suppose we are in a context where both the exact number of smart linguists and the exact number of smart semanticists is relevant. In such a context, when (59) is uttered without focus on *linguists*, it would normally implicate (60), but not (61). It seems then that in a given context, not all Horn scales associated with items in the sentence need be available for deriving an implicature. Our derivation mechanism is still not constrained enough.

Let us consider now (62) and (63).

(62) Some linguists are smart.

(63) Not all linguists are smart.

As we saw in section 1.4, <some, all> is not a Horn scale, because *some* and *all* do not have the same monotonicity properties. In order to derive (63) as an implicature of (62), we have to use the scale <some linguists, all linguists>.

Rooth (1992) points out that scalar implicatures depend on focus:

(64) He is GOOD at math.

Implicature: not wonderful at math

(65) He is good at MATH

Implicature: not good at other subjects

If focus determines the element to be compared, i.e. if we only look at Horn scales which include the focal element as a member, this might pose a problem for our analysis of (62). Consider the following example:

(66) SOME semanticists and ALL syntacticians have read at least one article by Chomsky.

When (66) is pronounced with a pitch accent on *some* and *all*, it is plausible to assume that the contrastive focus here is on the determiners and not on the DPs. The sentence clearly implicates that not all semanticists read an article by Chomsky. It also seems that in deriving the implicature we use the contrast that is made in the sentence between *some* and *all*. This is a problem since our mechanism disallows the scale <some, all>.

When discussing Horn scales it was mentioned that these scales are not necessarily based on pure entailment. The notion of entailment in a context was introduced. However, there are cases in which the relevant ‘scale’ (or set of alternatives) is not based on ‘contextual entailments’, but purely on world knowledge. (67) is such a case.

(67) A: What have you done with that mail?

B: I’ve typed it

Implicature: B has not mailed it yet.

The implicature that B has not mailed the letter yet is possible also in contexts where mailing a letter does not ‘contextually entail’ typing it, for example, when handwriting is an option too.

Another problem concerns the derivation of strong implicatures. Our process derives directly only weak implicatures. We already saw that in the case of the <or,and> scale it was impossible to derive the strong implicature from the weak implicature by simply adding an additional premise, hence some general strengthening principle must be stipulated. Unless a plausible pragmatic explanation for the strengthening process is found, there is no justification for relating the strong implicatures to the weak ones. It might as well be that the strong implicatures are derived completely separately from the weak implicatures.

Our theory is arbitrary in yet another way. We have assumed with Horn and Matsumoto that the pragmatic concept of ‘informativeness’ should be restricted by the formal notion of ‘monotonicity’. And if we make that assumption, aren’t we loosing the original Gricean communicative motivation? I cannot think of any language external explanation to why monotonicity should be involved at all in determining the conditions under which a speaker makes a ‘correct’ use of a sentence. It seems to me that the attempt to fix the Gricean theory ended in voiding it from its original insight. If this is the case wouldn’t it be better for us to look for a solution somewhere else?

In the next section I will present yet another problem for Gricean theories of Quantity implicatures.

1.7 ‘The projection problem’ for Scalar implicatures

1.7.1 ‘Embedded’ scalar implicatures

Landman (2000) points out a serious problem with Horn’s account of scalar implicature. Horn’s theory cannot work for cases where the scalar element is in the scope of another operator. Instances of this problem have been discussed in the literature before (Gazdar 1979, Hirschberg 1985, Horn 1989). Gazdar (1979) in his formalization of the Grice/Horn theory suggests to simply restrict the derivation of scalar implicatures to ‘simple’ sentences. So, Gazdar explicitly doesn’t have a theory of implicatures in complex sentences. But, as Landman points out, such a theory is needed, because logically complex sentences can have implicatures. The problem is that Horn’s theory, applied as is to complex sentences makes wildly incorrect predictions.

Consider for example an utterance of (68) as an answer to the following question: *For every x, how many children does x have?*

(68) Everyone has 3 children.

Horn would derive (69) as a conversational implicature of (68).

(69) S knows that [It is not the case that [everyone has 4 children]].

This is obviously the wrong implicature. (69) is equivalent to (70), which is too weak. The correct implicature should be (71):

(70) S knows that **someone** does not have 4 children.

(71) S knows that for every x, x does not have 4 children.

Landman proposes that the ‘core’ of the scalar implicature, *x does not have 4 children*, is derived at the earliest level in the grammatical derivation of the sentence asserted where an appropriate scale is available, and that the actual implicature of the sentence is built up, following its semantic composition. According to Landman, an implicature will inherit up, unless it contradicts the meaning of the sentence or is entailed by it. So, interestingly, Landman suggests that scalar implicatures involving numerical scales are not derived by some Gricean process, but **computed** by the grammar. Actually, Landman’s problem shows also in clearly context dependent cases, as shown in (72).

(72) A: Did everyone order beer?

B: Some ordered orange juice.

implicature: B knows that some did not order beer.

What is the prediction of the Gamut-Horn-Matsumoto derivation process for (72)? Given that <order orange juice, order orange juice and beer> is a Horn-scale, the process can derive only the weak implicature that B does not know that some ordered orange juice and beer. Adding the assumption that B is not agnostic about the truth of *Some ordered orange juice and beer*, we get:

(73) B knows that it is not the case that some ordered orange juice and beer. = B knows that no one ordered orange juice and beer.

This is wrong. The correct implicature should be (74):

(74) B knows that the latter (i.e. the ones that he mentioned) did not order orange juice and beer.

We have two problems here. The first is with the scope of negation, and the second, with the anaphoric reference of the implicature to the meaning. A detailed discussion of ‘embedded’ implicatures is given in chapter 5, section 1.

1.7.2 Suspension and cancellation of scalar implicatures

Levinson (1983, 2000) claims that scalar implicatures have a projection behavior that sometimes parallels the familiar behavior of presuppositions. (75)'s implicature that John doesn't have more than 3 children,'survives' in (76) and 'disappears' in (77) - (79).

(75) John has three children.

(76) It is possible that John has three children.

(77) John has three, if not four children.

(78) John has three or four children.

(79) John doesn't have three children.

Compare this data with the behavior of the presupposition of *stop*:

(80) John stopped smoking. (presupposes: John smoked)

(81) It is possible that John stopped smoking.

(82) John stopped smoking, if indeed he smoked.

(83) John stopped smoking, or he didn't smoke.

(84) John didn't stop smoking.

(80)'s presupposition 'survives' in (81) and (84), and 'disappears' in (82)-(83). Levinson (2000) invites researchers working in dynamic semantics to pursue this parallelism, and find an independent explanation for the alleged failure of implicatures under negation (Horn 1989 claims that scalar implicatures do not occur under downward entailing operators in general).

Kadmon (2001) argues that there is no real parallelism between the projection of non-presuppositional implicatures (such as scalar implicatures) and the ‘projection’ of presuppositions. The most promising theories of presupposition projection explain their projection behavior from the fact that a presupposition must be satisfied by its local context (see for example Karttunen 1974, Stalnaker 1974, Karttunen and Peters 1979, Heim 1983, Roberts 1989, Roberts 1996a, Beaver 1995).[2] Consider for example the case of conjunction, where the second conjunct has a presupposition, as the case in (85) and (86):

(85) John used to smoke, and he stopped smoking

(86) John took his doctor’s advice, and he stopped smoking.

The first conjunct plays a role in determining whether a presupposition of the second conjunct will be inherited by the whole conjunction. This makes sense – the second conjunct is added to a context which already includes the first conjunct. Hence, what is relevant for determining the whole conjunction’s presupposition is the *local* context of the second conjunct; this local context is derived from the original context by adding to it the information in the first conjunct. In example (85) above, the whole sentence does not inherit the second conjunct’s presupposition that John used to smoke, because this presupposition is already satisfied by its local context (it is entailed by the first conjunct). In example (86), the first conjunct does not entail the

[2] Simons 2001 offers an alternative account of presuppositions associated with verbs like *stop*.

second conjunct's presupposition, and in order for the local context to satisfy it, it must be included in the context of utterance of the whole sentence, i.e., presupposed by the whole sentence. Kadmon remarks that it doesn't make sense to talk about scalar implicatures as being "satisfied". No sensible notion of satisfaction exists for scalar implicatures – the information carried by a scalar implicature is part of the information that the speaker intends to communicate, not something which is taken for granted.

A different approach, which I'll adopt in this dissertation, is to explain cases of implicature 'inheritance' and 'disappearance' as following directly from the way they are computed. I'll discuss implicatures in downward entailing contexts in chapter 5 section 2, and cases where implicatures are 'suspended' (such as examples 78 and 79) in chapter 5, section 3.

In the following chapters I propose an alternative to the Gricean way of deriving Quantity implicatures, an alternative that avoids the problems of Gricean theories, and explains in a straightforward way the facts about implicature 'projection'.

Chapter 2

Exhaustivity, *Only* and Scalar Implicatures

In chapter 1 I presented the Gricean approach to the derivation of certain context dependent inferences which were labeled “Strong Quantity Implicatures” or “Scalar Implicatures”. In this chapter I’ll show that we can avoid many of the challenges to the Gricean approach if we adopt a different point of view. I’ll explore the possibility of analyzing these inferences as the result of the application of an exhaustivity operator, which is stipulated by Groenendijk and Stokhof (1984b), and can be thought of as an answerhood constraint within their theory of questions and answers.

2.1 A sketch of Groenendijk and Stokhof’s theory of questions and answers

Groenendijk and Stokhof (1984a, 1984b) take it that the denotation of a question in a world w_0, is a proposition which expresses the true and complete answer to that question in w_0. For example, the yes-no question, *Does John come?*, denotes in w_0 the proposition that John comes, if John comes in w_0 , and the proposition that John doesn’t come, if John doesn’t come in w_0. The one-place constituent question, *Who comes?*, will denote in w_0 - the proposition that only John, Bill and Sue come iff John, Bill and Sue are the only comers in w_0.

(1) Does John come? $\rightsquigarrow \lambda w[\mathrm{COME}(john, w) = \mathrm{COME}(john, w_0)]$

Who comes? $\rightsquigarrow \lambda w[\lambda x \mathrm{COME}(x, w) = \lambda x \mathrm{COME}(x, w_0)]$

According to Groenendijk and Stokhof, the denotations of questions are computed in two steps. Questions have underlying abstracts: a truth value in the case of yes-no questions, a set in the case of one-place constituent questions, and a relation in the case of two-place constituent questions. The abstract of *Does John come?* is 1, if John actually comes, and 0 if John doesn't come. The abstract of *Who comes?* is the set of comers, and the abstract of *Who kissed who?* is the set of ordered pairs of individuals in which the first kissed the second. A type shifting operation lifts the abstract to the proposition which is the denotation of the question.

(2) The abstract of *does John come?* in w_0 = $\mathrm{COME}(john,w_0)$

The abstract of *who comes?* in w_0 = $\lambda x \mathrm{COME}(x,w_0)$

The abstract of *who kissed who?* in w_0 = $\lambda x \lambda y \mathrm{KISS}(x,y,w_0)$

(3) $\mathrm{LIFT}[\alpha] = \lambda w[\alpha[w/w_0] = \alpha]$

where $\alpha[w/w_0]$ is the result of replacing the free occurrences of w_0 with w in α

(4) The extension of *does John come?* in w_0 = $\mathrm{LIFT}[\mathrm{COME}(john)]$ =

$\lambda w[\mathrm{COME}(john,w) = \mathrm{COME}(john,w_0)]$

The extension of *who comes?* in w_0 = $\mathrm{LIFT}[\lambda x \mathrm{COME}(x,w_0)]$ =

$\lambda w[\lambda x \mathrm{COME}(x,w) = \lambda x \mathrm{COME}(x,w_0)]$

The extension of *who kissed who?* in w_0 = LIFT[$\lambda x \lambda y$KISS(x,y,w_0)] =

$\lambda w[\lambda x \lambda y$KISS(x,y,w) = $\lambda x \lambda y$KISS(x,y,w_0)]

We get the intension of a question, by abstracting over the free variable w_0 in the extension. Thus the intension of a question is a function from possible worlds to propositions: that function which assigns to every evaluation world w_0, the proposition which is the true and complete answer to the question in w_0. We can think of the intension of a question as a set of propositions, the set of all possible complete answers to the question.

(5) The intension of *does John come?* = $\lambda w_0 \lambda w$[COME(john,w) = COME(john,w_0)]

The intension of *who comes?* = $\lambda w_0 \lambda w[\lambda x$COME(x,w) = λxCOME(x,w_0)]

The intension of *who kisses who?* = $\lambda w_0 \lambda w[\lambda x \lambda y$KISS(x,y,w) = $\lambda x \lambda y$KISS(x,y,w_0)]

A proposition that expresses a complete answer to a question in some world is a member of the intension of the question. A partial answer would be a proposition which is formed by a union of some propositions in the intension. For example, the proposition expressed by *only John came* is a complete answer to *who came?*, while the proposition expressed by *John came* is a partial answer.

(6) Let Q be a question intension (a set of propositions), and let p be a proposition,

1. p is a complete semantic answer to Q iff $p \in Q$
2. p is a partial semantic answer to Q iff $p \neq \emptyset$, and $\exists X \subset Q$: $p = \cup X$

One place constituent questions can be answered not only with sentences (which express propositions), but with other types of constituents as well. For example, the answer *nobody* is intuitively also a complete answer to the question *who came?*, and the answer *some men* is intuitively a partial answer to that question. The notion of semantic answerhood can be formulated for short answers as well.

(7) Let *Q* be a question such that its abstract denotes (in a world w_0) the set P, and its intension is Q, and let T be a generalized quantifier (a set of sets),

1. T is a complete short semantic answer to Q iff T(P) is defined and $\lambda w_0 T(P)$ is a complete semantic answer to Q.
2. T is a partial short semantic answer to Q iff T(P) is defined and $\lambda w_0 T(P)$ is a partial semantic answer to Q.

On Groenendijk and Stokhof's theory, intensions of questions form partitions on the set of possible worlds.

(8) A partition of W is a set P of subsets of W – called the blocks or cells of P, such that:

1. For every $B \in P$: $B \neq \emptyset$
2. $\cup\{B: B \in P\} = W$
3. for every $B_1, B_2 \in P$, if $B_1 \neq B_2$, then $B_1 \cap B_2 = \emptyset$

Intensions of questions are of the form $\lambda w_0 \lambda w[\alpha[w/w_0] = \alpha]$, i.e., they are relations between possible worlds. It is not hard to see that this type of relation is reflexive, symmetric and transitive – it is an equivalence relation on the set of possible worlds. An equivalence relation on some set defines a partition of that set. The blocks of the partition are formed from the members of the set which are related to each other. Let me demonstrate this with an example.

(9) Question: Who came?

Intension of the question: $\lambda w_0 \lambda w[\lambda x COME(x,w) = \lambda x COME(x,w_0)]$

Let us assume for simplicity that the domain of individuals, D, consists only of two individuals: John and Mary. There are four propositions which are potential complete answers to our question: no one came, only John came, only Mary came, John and Mary came. The set of possible worlds, W, can be partitioned along these four options. The potential complete answers are the blocks of the partition. This view of questions fits naturally with the fact that a question is a request for information. If a complete and true answer is given, then the utterer of the question eliminates all blocks on the partition but one. If a true, but incomplete answer is given (for example, *John came*), the utterer eliminates some blocks of the partition (in this case the blocks where nobody came and where only Mary came). In both cases, when the question is answered, the utterer's information increases: she locates herself in a smaller area of the range of possibilities.

Sometimes, not the whole range of possible answers is open to us. The utterer of *who came?* may know, for example, that it is not the case that Mary came. In this case, the answer *John came* serves (in the above example) as a complete answer to the question. To give another example, if someone asks whether she should take an umbrella, the reply *it's raining* serves as a complete answer in the context where one takes an umbrella if it's raining. We can define the notions of a complete or partial pragmatic answer in the following way:

(10) $\uparrow(I,p)$, the update of information I by proposition p, is $I \cap p$, if $I \cap p \neq \emptyset$. It is I, otherwise.

(11) p is a complete pragmatic answer to Q in I iff $\uparrow(I,p) \in Q$

p is a partial pragmatic answer to Q in I iff $\uparrow(I,p) \neq \emptyset$, and $\exists X \subset Q$: $\uparrow(I,p) = \cup X$

2.2 Groenendijk and Stokhof's exhaustivity operator and scalar implicatures

Groenendijk and Stokhof (1984b) observe that answers to questions are normally interpreted exhaustively. For example, the answer *Galit* to the question *Who is writing a PhD in semantics?* usually implies that no other student (in the linguistics department at Tel Aviv University) is writing a PhD in semantics . To account for this, they stipulate a semantic exhaustivity operator which relates the answer and the abstract underlying the question, and which is supposed to have the semantic effect of the word *only*. As we will see, many of the implicatures traditionally explained using Grice's first maxim of Quantity can be accounted for with this operator.

The operator *exh*, given in (12) below, is applied to a pair consisting of the abstract underlying the question asked - represented by the variable P - and the short answer given - represented by the variable T. It ensures that the given answer is true - John comes, and exhaustive - no one else comes. If the answer is given in the form of a proposition, the operator will apply to the focused part, for example to *John* in *[John]$_F$ comes*.

(12) exh = $\lambda T \lambda P[T(P) \wedge \neg \exists Q[T(Q) \wedge Q \subset P]]$

I'll illustrate how the exhaustivity operator works in a few examples. Later I discuss some examples where it fails.

(13) Who came?

[John and Mary]$_F$ came

P= ABS(who came?) $\rightsquigarrow \lambda x COME(x,w_0)$ (tense is being ignored)

T= john and mary $\rightsquigarrow \lambda P[P(j) \wedge P(m)]$

Exh(13) = $\lambda P[P(j) \wedge P(m)](\lambda x COME(x,w_0)) \wedge \neg \exists Q[\lambda P[P(j) \wedge P(m)](Q) \wedge Q \subset \lambda x COME(x,w_0)]]$ = $COME(j,w_0) \wedge COME(m,w_0) \wedge \neg \exists Q[Q(j) \wedge Q(m) \wedge Q \subset \lambda x COME(x,w_0)]]$

In words: John came and Mary came and there is no proper subset of the set of comers which includes John and Mary.

This means that only John and Mary came. Here's why. Exh(13) requires that John and Mary came. Let us assume that Harry came as well, so $\lambda x COME(x,w_0)$ = {John, Mary, Harry}. But then, {John, Mary} is a proper subset of $\lambda x COME(x,w_0)$, and this is not allowed by Exh(13). Hence $\lambda x COME(x,w_0)$ = {John, Mary}.

(14) Whom did John kiss?

John kissed $[\text{Sarah}]_F$

P= ABS(whom did John kiss?) $\rightsquigarrow \lambda y KISS(j,y,w_0)$

T= sarah $\rightsquigarrow \lambda P[P(s)]$

$$Exh(14) = KISS(j,s,w_0) \wedge \neg\exists Q[Q(s) \wedge Q \subset \lambda y KISS(j,y,w_0)]]$$

In words: John kissed Sarah and there is no proper subset of the set of persons kissed by John, which includes Sarah.

This means that John kissed only Sarah. If John kissed someone else as well, there would be a proper subset of individuals kissed by John that includes Sarah, like the set {Sarah}itself.

We see that in these cases, exhaustiveness accounts for the inferences which were supposed to be scalar implicatures derived via scales such as <John, John and Mary, John and Mary and Harry, …>. For the implicature theory to work, we had to stipulate that all elements on the scale had the same monotonicity properties. This stipulation was needed for excluding scales such as <John, only John>. If such a scale were allowed, there would be no reason why the sentence *John came* couldn't implicate that not only John came (see chapter 1, section 1.4). As discussed in chapter 1, section 1.6, the monotonicity stipulation is problematic. The exhaustivity operator gives the desired result without a monotonicity stipulation.

Before we continue to look at more examples, I would like to state explicitly what it is that I'm proposing about scalar implicatures. I suggest that scalar implicatures are not implicatures (in the Gricean sense) at all. The inferences which are traditionally analyzed as scalar implicatures of a certain sentence, *A*, are merely entailments of *exh(A, Q)*, where *A* is taken to be a complete or partial semantic answer to some question *Q* (*Q* may be explicit or implicit).

I assume that question-answer pairs are systematically ambiguous between exhaustive and non-exhaustive interpretations, i.e., interpretations where *exh* applies and interpretations where *exh* doesn't apply. Let us look at a few examples:

(15) A: Who came?
B: Bill, Mary and Sue
A: Why didn't Sarah come?

(16) A: Who came?
B: Bill, Mary and Sue
A: And no one else? / Did anyone else come?

(17) A: Who came?

B: Bill, Mary and Sue

A: And who else?

All the above short conversations are natural, and they show that *A* has the option to interpret *B*'s answer exhaustively (15 above), non-exhaustively (17 above), or wonder whether B's answer should be interpreted exhaustively or non-exhaustively (16 above).

I take it that *exh* is a **semantic** operator, which strengthens the meaning of the answer it operates on in a certain way. This means that the implicature effect is really analyzed as an ambiguity. The stronger meaning of *A* is semantic - it is the effect of *exh(A, Q)*. The inferences in question are not derived by assuming that stronger propositions than the one uttered are false, they are due to the semantic operator of exhaustivity.

Grice's (1989) point against an ambiguity theory for the kind of inferences he labeled 'generalized conversational implicatures' is what he calls "Modified Occam's Razor" - "senses are not to be multiplied beyond necessity". What Grice had in mind was that we should not stipulate 'unnecessary' multiple meaning for lexical items. For example, it's preferable to assume that the connective *or* has only an inclusive meaning (it is not ambiguous between an inclusive and an exclusive meaning). The exclusive interpretation should be derived from the inclusive meaning by some general pragmatic principles (though we saw in chapter 1 that Gricean theories have great difficulties in succeeding in this task).

The theory proposed here does not go against Grice's "Modified Occam's Razor" . I do not suggest that scalar implicatures are due to idiosyncratic ambiguities of certain lexical items. The source of the ambiguity in question is the optionality of the application of *exh*. I do not even stipulate a special sense for *exh* – I assume it has the meaning of the word *only* (though, as we will see later, Groenendijk and Stokhof's semantics for *exh* is not good enough).

Although the approach sketched here is essentially very different from the Gricean theory, the main notions of the Gricean theory can be (partially) translated into it. Let us restrict ourselves, for the moment, to explicit question-answer pairs. We start by defining the notion of an implicature.

(18) A implicates B in the context of a question Q, if exh(A, Q) entails B, and A does not entail B.

We preserve the context dependency and cancelability of implicatures. A sentence may have or may not have a certain implicature depending on which question it answers. What looks like canceling an implicature is merely weakening back from exh(A, Q) to A. I delay the discussion on explicit 'suspension' and 'cancellation' of implicatures to chapter 5, section 3.

As already mentioned, I take *exh* to be a semantic operator with the meaning of *only*. This doesn't mean that pragmatic factors are not involved. Obviously, *exh* is a very useful operator. It adds information to the meaning of the answer, and as we saw in examples (13) and (14) above, its application sometimes turns partial semantic answers into complete semantic answers. Now, the word *only*, in virtue of its meaning, is a very convenient device in explicitly expressing complete semantic answers. I think it is not far-fetched to assume that users of language developed a certain convention about question answering; a convention according to which an answer can be or should be interpreted as implicitly containing *only* (in a position determined by the question). In Gricean terms: one may stipulate a **Quality** maxim for question answering: “Answer the question!”. But “Answer the Question!” on Groenendijk and Stokhof’s theory, means: give a **true and complete** answer to the question. This completeness is part of the Quantity maxim in Gricean theories, but in the theory suggested here, it is part of **Quality** (it is part of the definition of a true answer).

Now, the meanings of sentences are often too **weak** to provide an answer that satisfies this strong Quality constraint. The assumption that nevertheless the answerer obeys the Quality maxim brings in the assumption that she meant her statement to be **stronger** than it strictly speaking is. If we assume that the hearer and the speaker know that an utterance of φ **can** be interpreted as an implicit utterance of $exh(\varphi)$, the hearer can assume, with the maxim of **Quality** that the speaker’s utterance should be interpreted as $exh(\varphi)$. The basic assumption to make this possible is that indeed the **semantics** allows systematically interpretations strengthened with *exh*. That is: φ **can** be interpreted as φ, but also as $exh(\varphi)$.

I cannot offer here a theory of how we choose between the strong and weak interpretations of answers. My intuitions are that given a question answer pair, we tend to interpret the answer exhaustively (though it should be stressed that exhaustiveness itself is, often, a context-dependent, or context restricted notion. See discussion below).

For the record, let me be precise here about the relation between Groenendijk and Stokhof's theory and what I propose. In Groenendijk and Stokhof's theory *exh* is a semantic operation which is part of the meaning of a statement **as** an answer to the question. My view is slightly different, though this difference may be merely cosmetic. I assume that *exh* is a semantic operation which can be part of the meaning of a sentence, even if not lexically expressed. So *exh* **itself** is not brought in by the question-answer relation as it is in Groenendijk and Stokhof's theory, but is **triggered** by the latter by the maxim of Quality.

The **core** of my proposal – and this is very different form Groenendijk and Stokhof's theory, is the strong interpretation of the maxim of Quality in the context of question-answer pairs. My proposal is that an answer to a question which is not semantically complete violates Quality. Hence the semantics **must** provide a meaning for the sentence which doesn't violate Quality (more about this in chapter 6). Groenendijk and Stokhof make a distinction between a semantic answer (a true and complete answer) and a pragmatic answer (an answer that may stand in a weaker relation). Crucially, then, I don't follow Groenendijk and Stokhof here, and as we will see, that makes all the difference.

Another point which I'll have to address is that there are quite a bit more contexts than those where there is an explicit question available where exhaustiveness is triggered. In absence of an explicit question or prosodically marked focus – which according to Roberts' focus theory (Roberts 1996b), provides clues towards questions implicit in the discourse which the sentence answers – a certain sentence may have various possible strengthenings (i.e. we could take it as an answer to different questions). I will turn to this point in chapter 7.

2.3 Groenendijk and Stokhof's exhaustivity operator in complex examples

Let us now see how Groenendijk and Stokhof's exhaustivity operator fairs in more complicated examples.

(19) Who came?

[John or Mary]$_F$ came.

P= ABS(who came?) $\rightsquigarrow \lambda x COME(x,w_0)$

T= john or mary $\rightsquigarrow \lambda P[P(j) \vee P(m)]$

Exh(19) = $COME(j,w_0) \vee COME(m,w_0) \wedge \neg\exists Q[[Q(j) \vee Q(m)] \wedge Q \subset \lambda x COME(x,w_0)]]$

In words: John came or Mary came and there is no proper subset of comers, which includes John or which includes Mary.

This means that only John came or only Mary came. If both came, we could find a proper subset of comers which includes John, namely {John}.

The meaning of exh(19) can be split into 3 components:

1. John or Mary came
2. It is not the case that both John and Mary came
3. No one who is not John or Mary came

The first component is simply *John or Mary came* without exhaustivization. According to definition (18), both 2 and 3 are implicatures of *John or Mary came* in the context of *who came?*. This seems right for some contexts where (19) is used, but too strong for others – there are contexts where the use of *John or Mary came* as an answer to *who came* implicates 2 but not 3. How can we get this 'intermediate' strengthening? One simple way is to assume that the abstract of *who came?* can contain some contextual restriction, C. This means that the question *who came?* is understood as *which C's came?* In such cases we don't expect the listener to specify all comers, but only a contextually relevant subset of comers.[1]

(20) Who came?

John or Bill.

P= ABS(who came?) $\rightsquigarrow \lambda x[C(x,w_0) \wedge COME(x,w_0)]$

T= john or bill $\rightsquigarrow \lambda P[P(j) \vee P(b)]$

[1] For a discussion on domain selection see for example Westerståhl 1984 and von Fintel 1994.

$Exh(20) = [[C(j,w_0) \wedge COME(j,w_0)] \vee [C(b,w_0) \wedge COME(b,w_0)]] \wedge \neg\exists Q[[Q(j) \vee Q(b)] \wedge Q \subset \lambda y[C(y,w_0) \wedge COME(y,w_0)]]$

In words: John is an individual with property C who came or Bill is an individual with property C who came, and there is no proper subset of the set of individuals with property C who came, which includes John or which includes Bill. I.e. The only individual with property C who came is John or the only individual with property C who came is Bill.

The answer presupposes that both John and Bill have the property C, and hence exh(20) retains the implicature 2, but not the implicature 3.

I think that implicature 3 above is more common in cases where the question includes an explicit restriction as in (21):

(21) Which pets does Mary have?
Mary has a snake or an iguana.

I think that the answer in (21) clearly implicates that Mary doesn't have a cat.

It is important to note that in the exhaustiveness analysis, the implicature that Mary doesn't have a cat is linked to the implicature that Mary doesn't have a snake and an iguana. We can explain the cases where both are present, and we can explain the cases where only the latter is present, by assuming a contextual restriction on the question. Exhaustivization forces an exclusive interpretation for *or*. I think that this is basically a good prediction. If the question in (21) is answered with an explicit use of *only*, *Mary has only a snake or an iguana*, we understand that Mary has only one pet. Note that *Mary has only a snake or an iguana and maybe both* sounds contradictory. However, I think that sometimes the answer in (21) can also be interpreted as conveying that Mary has a snake or an iguana and maybe both, but not other pets, and this fact remains unexplained so far. The interpretation that we need for this case is *Mary has only [a snake or an iguana or both]*. I'll discuss this further in chapter 3.

We see that exhaustivity predicts in a straightforward way the exclusive interpretation of *or*. Hence we don't need to worry about the problem Gricean pragmatics has in excluding scales such as <a or b but not both, a or b> (see discussion in chapter 1, sections 1.3.2 and 1.4).

This analysis also helps us to solve a problem that Gricean pragmatics has with the implicature of sentences of the form *A or B or C*, a problem that I didn't mention so far. Let us consider B's answer to A's question in (22) below:

(22) A: Who kissed Sarah?

B: John or Bill or Fred.

Horn (1972) would predict that (22B) implicates the following:

(23) It is not the case that John and Bill and Fred kissed Sarah.

This is too weak. The right implicature is:

(24) Only John or only Bill or only Fred kissed Sarah

The exhaustiveness analysis gets the correct inference without a problem.

P= ABS(who kissed sarah?) $\leadsto \lambda x KISS(x,s,w_0)$

T= john or bill or fred $\leadsto \lambda P[P(j) \vee P(b) \vee P(f)]$

Exh(22) = $[KISS(j,s,w_0) \vee KISS(b,s,w_0) \vee KISS(f,s,w_0)] \wedge \neg \exists Q[[Q(j,s) \vee Q(b,s) \vee Q(f,s)] \wedge Q \subset \lambda x KISS(x,s,w_0)]]$

In words: John kissed Sarah or Bill kissed Sarah or Fred kissed Sarah and there is no subset of Sarah-kissers, which includes John or Bill or Fred. I.e. only John or only Bill or only Fred kissed Sarah.

The following is another example where the exhaustivity operator fairs better than the implicature theory.

(25) Who comes?

Some man

P= ABS(who comes?) $\leadsto \lambda x COME(x,w_0)$

T= some man $\leadsto \lambda P[\lambda x MAN(x,w_0) \cap P \neq \emptyset]$

$exh(25) = \lambda xMAN(x,w_0)\cap\lambda xCOME(x,w_0)\neq\varnothing \wedge\neg\exists Q[MAN(w_0)\cap Q\neq\varnothing \wedge Q \subset \lambda xCOME(x,w_0)]$

In words: Some man comes and there is no proper subset of comers which are men

Exh(25) means that one man comes, and nobody else comes. Suppose that John is a man that comes. If Harry or Mary came as well, there would be a proper subset of comers which are men – the set {John}.

The effect of exhaustivization in this example is twofold: exactly one man comes, and only men come. According to definition (18), both are implicatures of (25). Furthermore, in a context where there is more than one man, we get an additional inference that not all men come. The latter inference is a classical implicature, traditionally accounted for via the <some, all> scale. The exhaustiveness analysis predicts this inference without extra work, since in most contexts the assumption that there is more than one man is warranted. We also avoid the wrong prediction that the answer *some man comes* implicates that (the speaker knows that) John doesn't come – an implicature that could, in principle, be derived using the scale <some man, John>. The latter is a non-existent implicature which shows we somehow would have to block the scale <some man, John>.

If we assume a contextual restriction, C, on the question, we predict a weaker implicature for (25):

$P= ABS(\text{who comes?}) \rightsquigarrow \lambda x[C(x,w_0)\wedge COME(x,w_0)]$

T= some man $\rightsquigarrow \lambda P[\lambda x MAN(x,w_0) \cap P \neq \varnothing]$

exh(25’) = $\lambda x MAN(x,w_0) \cap \lambda x[C(x,w_0) \wedge COME(x,w_0)] \neq \varnothing \wedge \neg \exists Q[MAN(w_0) \cap Q \neq \varnothing \wedge Q \subset \lambda x[C(x,w_0) \wedge COME(x,w_0)]]$

In words: Some man with a property C comes and there is no proper subset of comers with property C which are men.

Exh(25’) is weaker than exh(25). It means that only one man with property C comes, and that no one else with property C comes. Exh(25’) allows other comers as long as they don’t have some contextual property C.

Groenendijk and Stokhof (1984b) show how the biconditional interpretation of a conditional is predicted, if an intensional version of the exhaustiveness operator is used.

(26) Does John come?

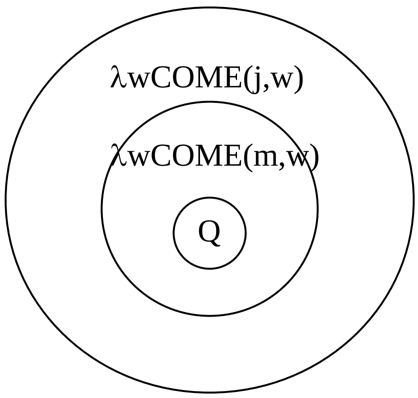
If Mary comes.

P= ABS(does john come?) $\rightsquigarrow \lambda w COME(j,w)$ (the set of worlds where John comes)

T= if mary comes $\rightsquigarrow \lambda P[\lambda w COME(m,w) \subseteq P]$; P is a variable of type $<s, <e,t>>$

Exh(26) = $[\lambda w COME(m,w) \subseteq \lambda w COME(j,w)] \wedge \neg \exists Q[\lambda w COME(m,w) \subseteq Q) \wedge Q \subset \lambda w COME(j,w)]]$

Exh(26) is equivalent to the biconditional *John comes if and only if Mary comes*. It is easy to see this, if we look at diagram (A) below. The first main conjunct of exh(26) requires that the set of worlds λwCOME(m,w) is a subset of λwCOME(j,w). The second main conjunct of exh(26) disallows the existence of a set of worlds Q which is both a proper subset of λwCOME(j,w) and a superset of λwCOME(m,w). This means that λwCOME(m,w) itself cannot be a proper subset of λwCOME(j,w), hence it must be identical to it. But these are exactly the truth conditions of the biconditional.



A

Let us turn now to some cases with numerals. The question-answer pair in (27), with the focus on *John*, and not on *three,* is a typical case where a numeral has an 'at least interpretation'. Groenendijk and Stokhof's exhaustivity operator predicts correctly that (27) does not implicate that John has at most 3 chairs.

(27) Who has 3 chairs (that I can borrow)?

[John]$_F$ has 3 chairs.

P= ABS(who has 3 chairs?) $\rightsquigarrow \lambda x|\lambda y[CHAIR(y,w_0) \wedge HAVE(x,y,w_0)]| \geq 3$

T= john $\rightsquigarrow \lambda P[P(j)]$

Exh(27) = $|\lambda y[CHAIR(y,w_0) \wedge HAVE(j,y,w_0)]| \geq 3 \wedge \neg\exists Q[Q(j) \wedge Q \subset$

$\lambda x|\lambda y\ [CHAIR(y,w_0) \wedge HAVE(x,y,w_0)]| \geq 3|]$

In words: John has (at least) 3 chairs and there is no proper subset of possessors of (at least) 3 chairs, which includes John i.e. only John has (at least) 3 chairs.

Exh(27) means that only John has at least 3 chairs; it means that nobody else has 3 or more chairs, but it doesn't mean that John has exactly 3 chairs.
In (28) it is the number that is focused. It is a typical case where a numeral is believed to trigger a scalar implicature.

(28) How many chairs does John have?
John has $[3]_F$ chairs.

Groenendijk and Stokhof (1984b) did not deal with questions of the form *how many?*. I assume that a question such as *How many chairs does John have?* means something like *Which number(s) is/are such that John has at least that many chairs?* If John has, for example, exactly 3 chairs, then *one*, *two* and *three* are all such numbers. I'll show that exhaustivity gives us the 'exactly interpretation' of the number given as the answer. Assuming the above interpretation for the question, the abstract underlying it, is a set of numbers such that John has at least that number of chairs. In analogy to the *who* questions, where the short answers were taken to be generalized quantifiers rather than individuals, I assume that the answer, *three*, is a generalized quantifier over sets of numbers - the set of sets of numbers that contain the number 3.

(28) How many chairs does John have?

John has $[3]_F$ chairs.

P = ABS(how many chairs does john have?) $\rightsquigarrow \lambda n|\lambda y[CHAIR(y,w_0) \wedge HAVE(j,y,w_0)]| \geq n$

T = three $\rightsquigarrow \lambda P[P(3)] = \lambda P \exists n[n = 3 \wedge P(n)]$

$exh(28) = |\lambda y[CHAIR(y,w_0) \wedge HAVE(j,y,w_0)]| \geq 3 \wedge \neg\exists Q[Q(3) \wedge Q \subset \lambda n|\lambda y[CHAIR(y,w_0) \wedge HAVE(j,y,w_0)]| \geq n]$

In words: John has at least 3 chairs, and there is no proper subset of numbers of chairs owned by John which contains 3.

This means that the set of numbers of chairs owned by John is {1, 2, 3}. I.e. that John has exactly 3 chairs. The first main conjunct of exh(28) requires that John has at least 3 chairs, the second main conjunct requires that John has no more than 3. If John had, for example, 4 chairs, then the set of numbers of chairs owned by John would be {1, 2, 3, 4}. In that case, there would be a proper subset containing 3, for example, the set {1, 2, 3}.

One might prefer (with Rullmann 1985) a question such as *how many chairs does John have?* to be interpreted as *Which number is such that John has exactly that many chairs?* rather than *Which number(s) is/are such that John has at least that many* *chairs?* I chose the latter for generality considerations. It is possible to analyze the *how many* cases exactly in the same way as the *who*/*which* cases; i.e. to let exhaustiveness come from the question/answer relation. There is no need to assume that exhaustiveness is explicitly built into the meaning of *how many* questions.

So far Groenendijk and Stokhof's analysis went very well. I now discuss two cases where the analysis does not make correct predictions. The first problem with the above analysis is that it wrongly predicts an 'exactly interpretation' also for (29).

(29) How many chairs does John have?

John has [at least 3] $_F$ chairs.

P = ABS(how many chairs does John have?) $\rightsquigarrow \lambda n|\lambda y[CHAIR(y,w_0) \wedge HAVE(j,y,w_0)]| \geq n$

T = at least three $\rightsquigarrow \lambda P\exists n[n \geq 3 \wedge P(n)]$

exh(29) = $|\lambda y[CHAIR(y,w_0) \wedge HAVE(j,y,w_0)]| \geq 3 \wedge \neg\exists Q[\exists n[n \geq 3 \wedge Q(n)] \wedge Q \subset \lambda n|\lambda y[CHAIR(y,w_0) \wedge HAVE(j,y,w_0)]| \geq n]$

In words: John has at least 3 chairs, and there is no proper subset of numbers of chairs owned by John which contains a number larger or equal to 3.

Exh(29), like exh(28) is also equivalent to 'John has exactly 3 chairs'. The first main conjunct of exh(29) ensures that John has at least 3 chairs. Unfortunately, the second main conjunct of exh(29) requires that John doesn't have more than 3 chairs. This is, of course, a wrong prediction.

A second case where Groenendijk and Stokhof's exhaustivity fails is the plural (30).

(30) Who came?

Some men came

Before working out this example, a brief presentation of the basic concepts of plurality theory is in order (see Scha 1981, Link 1983, Landman 1996).

The domain of individuals, D, includes both singular and plural entities, structured by a **part-of** relation, $\sqsubseteq$, "plural part-of", which is a partial order on D. (In fact we follow Link 1983 and Landman 2004 in assuming that $\sqsubseteq$ is not just any partial order, but a partial order that gives D the structure of a complete atomic Boolean algebra. See the works mentioned for details).

For $\forall$a, b$\in$D, D contains a plural individual, $a \sqcup b$, "**the sum of a and b**" (or the **join** of a and b), which is the smallest plural individual of D such that $a \sqsubseteq a \sqcup b$ and $b \sqsubseteq a \sqcup b$. For $\forall$a, b$\in$D, D contains a plural individual, $a \sqcap b$, "**the meet of a and b**", which is the biggest plural individual of D such that $a \sqcap b \sqsubseteq a$ and $a \sqcap b \sqsubseteq b$. D is closed under sum and meet formations. We assume that D contains an "improper individual" 0, which is the minimum of D: for $\forall d \in D$, $0 \sqsubseteq d$.

For every non-empty $X \subseteq D$, $\sqcup X$, "**the sum of X**", is the unique element (if there is such a unique element) that is the smallest element of D such that $\forall x \in X$: $x \sqsubseteq \sqcup X$.

Let $P \subseteq D$, $\sigma(P) = \sqcup P$ if $\sqcup P \in P$, undefined otherwise (This analysis of σ goes back to Sharvy 1980. σ is taken to be the interpretation of the definite article *the*).

$a \in D$ is an **atom** iff for every $b \in D$, if $b \sqsubseteq a$, then b=0 or b=a. **ATOM** is the set of atoms in D.

Let $P \subseteq D$, $*P = \{d \in D: \exists X \subseteq P: d = \sqcup X\}$ (closure under sum). For example, if $P=\{a,b,c\}$; $*P=\{a,b,c, a\sqcup b, a\sqcup c, a\sqcup b\sqcup c\}$. The * operator is taken to be pluralization.

Let $d \in D$, $ATOM(d) = \{a \in ATOM: a \sqsubseteq d\}$, $|d| = |ATOM(d)|$

Let us return now to the plural example:

(30) Who came?

Some men came

P= ABS(who came?) $\leadsto \lambda x{*}COME(x,w_0)$ = the set of singular and plural individuals in the set of all sums of coming individuals. Note that x ranges over singular and plural individuals

T = some men $\leadsto \lambda P[\exists x \in \lambda x{*}MAN(x,w_0): |x| \geq 1 \wedge P(x)]$; P is a variable ranging over sets of the form *X for some $X \subseteq ATOM$

$$exh(30) = [\exists x \in \lambda x{*}MAN(x,w_0): |x| \geq 1 \wedge \lambda x{*}COME(x,w_0)] \wedge \neg\exists Q[[\exists x \in \lambda x{*}MAN(x,w_0): |x| \geq 1 \wedge Q(x)] \wedge [Q \subset \lambda x{*}COME(x,w_0)]]$$

In words: One or more men came. There is no proper subset of the set containing the singular and plural comers that contains one or more men.

Like the singular case, discussed in (25) p. 62 above, exh(30) also means that only one man came. Here’s why. Let us assume that both John and Harry came, and no one else. In that case, $\lambda x{*}COME(x,w_0)$ = {John, Harry, John⊔Harry}. But then {John} is a proper subset containing singular or plural men, and exh(30) is false. Hence, exh(30) entails that only one man came. This, of course, is the wrong prediction: *Some men came* (as an answer to *who came?*) doesn’t have such an implicature.

The first challenge to an exhaustivity based implicature theory is to come up with the correct semantics of the exhaustivity operator, a semantics that will produce the desired inferences. Such a formulation will be suggested in chapter 3, but before that, I end this chapter by discussing briefly Bonomi and Casalegno’s (1993) semantics of *only*.

2.4 Bonomi and Casalegno's semantics of *only*

Bonomi and Casalegno (1993) suggest the following event based semantics for *only*:

(31) Let x be a variable of type d (the type of individual and plural entities), let e, f and g be variables of type e (the type of events), let F be a variable of type <d,<e,t>>, let Q be a variable of type <<d,<e,t>>, <e,t>>, and let $\subseteq_E$ be the inclusion relations between events.

$$\text{ONLY} = \lambda Q\lambda F\lambda e[Q(F)(e)\wedge\forall f[\exists xF(x)(f) \rightarrow\exists g[Q(F)(g)\wedge f \subseteq_E g]]]$$

Let me demonstrate this with an example.

(32) Only [John]$_F$ cried

John $\leadsto\lambda F\lambda eF(j)(e)$

cried $\leadsto\lambda x\lambda e[\text{CRIED}(e) \wedge \text{AG}(e,x)]$

only John $\leadsto$ONLY(j) $= \lambda Q\lambda F\lambda e[Q(F)(e)\wedge\forall f[\exists xF(x)(f) \rightarrow\exists g[Q(F)(g)\wedge f \subseteq_E g]]]$

$(\lambda F\lambda eF(j)(e)) = \lambda F\lambda e[F(j)(e)\wedge\forall f[\exists xF(x)(f) \rightarrow\exists g[F(j)(g)\wedge f \subseteq_E g]]]$

only John cried $\leadsto\lambda e[[\text{CRIED}(e)\wedge\text{AG}(e,j)] \wedge \forall f[\exists x[\text{CRIED}(f) \wedge \text{AG}(f,x)$

$\rightarrow\exists g[\text{CRIED}(g)\wedge\text{AG}(g,j) \wedge f \subseteq_E g]]]$

The meaning of the sentence is derived by applying existential closure to this set of events, and we get:

only John cried $\rightsquigarrow$ $\exists e[[\mathrm{CRIED}(e) \wedge \mathrm{AG}(e,j)] \wedge \forall f[\exists x[\mathrm{CRIED}(f) \wedge \mathrm{AG}(f,x)]$

$\rightarrow \exists g[\mathrm{CRIED}(g) \wedge \mathrm{AG}(g,j) \wedge f \subseteq_E g]]]$

In words: There is a crying event, e, whose agent is John, and for every crying event, f, whose agent is x, there is a crying event, g, whose agent is John, which includes f.

(32) means that only John cried. If Bill cried as well, there would be an event of crying whose agent is Bill, which does not include an event of crying whose agent is John.

Bonomi and Casalegno's *only*, unlike Groenendijk and Stokhof's exhaustivity operator gives the correct results in the case of plural NP's.

(33) Only [one or more boys]$_F$ cried

one or more boys $\rightsquigarrow \lambda F \lambda e \exists x[x \in {}^*\mathrm{BOY} \wedge F(x)(e)]$

cried $\rightsquigarrow \lambda x \lambda e[\mathrm{CRIED}(e) \wedge \mathrm{AG}(e,x)]$

Only one or more boys $\rightsquigarrow$ $\lambda Q \lambda F \lambda e[Q(F)(e) \wedge \forall f[\exists x F(x)(f) \rightarrow \exists g[Q(F)(g) \wedge f \subseteq_E g]]]$

$(\lambda F \lambda e \exists x[x \in {}^*\mathrm{BOY} \wedge F(x)(e)]) = \lambda F \lambda e[\exists x[x \in {}^*\mathrm{BOY} \wedge F(x)(e)] \wedge \forall f[\exists x F(x)(f)$

$\rightarrow \exists g[\exists x[x \in {}^*\mathrm{BOY} \wedge F(x)(g)] \wedge f \subseteq_E g]]]$

Only one or more boys cried $\rightsquigarrow \lambda e[\exists x[x \in {}^*BOY \wedge CRIED(e) \wedge AG(e,x)] \wedge$

$\forall f[\exists x[CRIED(f) \wedge AG(f,x) \rightarrow \exists g[\exists x[x \in {}^*BOY \wedge CRIED(g) \wedge AG(g,x)] \wedge f \subseteq_E g]]]$

After existential closure we obtain:

Only one or more boys cried $\rightsquigarrow \exists e[\exists x[x \in {}^*BOY \wedge CRIED(e) \wedge AG(e,x)] \wedge$

$\forall f[\exists x[CRIED(f) \wedge AG(f,x) \rightarrow \exists g[\exists x[x \in {}^*BOY \wedge CRIED(g) \wedge AG(g,x)] \wedge f \subseteq_E g]]]$

In words: There is crying event, e, whose agent is a plural individual of one or more boys, and for every crying event, f, whose agent is x, there is a crying event, g, whose agent is a plural individual of one or more boys, which includes f.

(33) means that only boys cried. If a girl cried as well, there would be an event of crying whose agent is a girl, which does not include an event of crying whose agent is a boy.

Bonomi and Casalegno offer a more complicated analysis to deal with cases where the focused element is something other than an NP. Let us see how they deal with a case of a number focus.

(34) Only $[\text{two}]_F$ boys cried

Bonomi and Casalegno suggest that an expression α of natural language should be translated to a pair <A,B> of expressions of the logical language. If α does not contain a focused element, then A=B, and each of them is simply the ‘normal’ translation of the expression. If α contains a focused element, $A \neq B$. B is the 'normal' translation, while A is the 'skeleton' of the appropriate category. Bonomi and Casalegno specify a 'skeleton' for each category. (This idea is used in Krifka 1991 for a recursive definition of structured meanings. A structured meaning of a sentence is pair whose members are (i) the property obtained by λ-abstracting on the focus, and (ii) the ordinary semantic interpretation of the focus. The structured meaning theory of focus was developed in von Stechow 1981, Klein and von Stechow 1982, Jacobs 1983 and von Stechow 1989).

two $\rightsquigarrow <\lambda X\lambda F\lambda e\exists x[x \neq y \wedge ATOM(x,X) \wedge ATOM(y,Y) \wedge F(x\cup_d y)(e)],$

$\lambda X\lambda F\lambda e\exists x\exists y[x \neq y \wedge ATOM(x,X) \wedge ATOM(y,Y) \wedge F(x\cup_d y)(e)]>$

X is a variable of type <d,t>

$[two]_F \rightsquigarrow <\lambda X\lambda F\lambda e[X(v) \wedge F(v)(e)],$

$\lambda X\lambda F\lambda e\exists x\exists y[x \neq y \wedge ATOM(x,X) \wedge ATOM(y,Y) \wedge F(x\cup_d y)(e)]]>$

The first element in the pair is the 'skeleton' of the category of determiners. v is a variable of type d.

boys $\rightsquigarrow$ <*BOY, *BOY>

cried $\rightsquigarrow <\lambda x\lambda e[CRIED(e) \wedge AG(e,x)], \lambda x\lambda e[CRIED(e) \wedge AG(e,x)]>$

Applying *boys* to *[two]*$_F$ gives:

$[\text{two}]_F$ boys $\rightsquigarrow$ $<\lambda F\lambda e[{*}BOY(v) \wedge F(v)(e)],$

$$\lambda F\lambda e\exists x\exists y[x{\neq}y \wedge ATOM(x,{*}BOY) \wedge ATOM(y,{*}BOY) \wedge F(x{\cup_d}\, y)(e)]]>$$

This can be abbreviated to:

$<\lambda F\lambda e[{*}BOY(v) \wedge F(v)(e)],$

$$\lambda F\lambda e\exists x[TWO\text{-}BOYS(x) \wedge F(x)(e)]]>$$

The new rules for *only* are as follows:

(35) If A and B are expressions of type <e,t>, then

ONLY<A,B> = $\lambda e[B(e){\wedge}\forall f[A^*(f) \rightarrow \exists g[B(g){\wedge} f \subseteq_E g]]]$, where *A is the existential closure of A.

If A and B are expressions of type <a,b>, where b is a 'normal' type (<e,t> is a 'normal' type; if β is a normal type, then $<\alpha,\beta>$ is a 'normal' type), then ONLY<A,B> = λX.ONLY<A(X), B(X)>

The translation of an expression of the form *only* α is <ONLY<A,B>, ONLY<A,B>>, where <A,B> is the translation of α.

Applying *only* to *[two]$_F$ boys* yields in a pair $<\alpha, \alpha>$ where α is equal to:

$\lambda F.ONLY<\lambda e[BOY(v) \wedge F(v)(e)], \lambda e\exists x[TWO\text{-}BOYS(x) \wedge F(x)(e)]]> =$

$\lambda F\lambda e[\exists x[\text{TWO-BOYS}(x) \wedge F(x)(e)] \wedge \forall f[\exists v[{*}\text{BOY}(v) \wedge F(v)(f)] \rightarrow \exists g[\exists x[\text{TWO-BOYS}(x) \wedge F(x)(g) \wedge f \subseteq_E g]]]$

Now we can apply CRIED, and existential closure on the set of events, and get:

$\exists e[\exists x[\text{TWO-BOYS}(x) \wedge \text{CRIED}(e) \wedge \text{AG}(e,x)] \wedge \forall f[\exists x[{*}\text{BOY}(x) \wedge \text{CRIED}(f) \wedge \text{AG}(f,x)] \rightarrow \exists g[\exists x[\text{TWO-BOYS}(x) \wedge \text{CRIED}(g) \wedge \text{AG}(g,x) \wedge f \subseteq_E g]]]$

In words: two boys cried, and every event of crying whose agent is a plural individual of one or more boys is included in an event of crying whose agent is a plural individual of two boys.

This means that exactly two boys cried. If more than two cried, there would be an event of crying whose agent is more than 2 boys, and that event would not be included in an event of crying whose agent is a plural individual of two boys.

Although Bonomi and Casalegno's ONLY works for many cases, it does not always give the correct result. For example consider:

(36) Only $[\text{some}]_F$ boys cried

(36) strongly suggests (even maybe entails) that not all boys cried. I won't give the derivation for this example, it is very similar to the derivation of (34). The reader can check for herself that what Bonomi and Casalegno get for (36) is the same as for *Only [one or more]$_F$ boys cried*: One or more boys cried, and every

event of crying whose agent is a plural individual of one or more boys is included in an event of crying whose agent is a plural individuals of one or more boys. But the two clearly differ.

Bonomi and Casalegno mention exhaustiveness, and stipulate an answerhood operator ANS, that takes scope over answers. They suggest that the translation of ANS(α) is ONLY$\langle A,B \rangle$, where $\langle A,B \rangle$ is the translation of α. Although Bonomi and Casalegno's ONLY might be a promising starting point for formulating the semantics of the exhaustivity operator, I find it too complicated for my taste. Moreover, I'm not sure if and how it could be modified to handle example (36) and other cases (such as conditional sentences like example (26) in the previous section). In the next chapter I suggest a simpler semantics for the exhaustivity operator which is more in the style of Groenendijk and Stokhof, but one that works for a much wider range of cases.

Chapter 3

Exhaustivity on the Domain of Singular and Plural Entities

In this chapter and the next, I suggest reformulations of the exhaustivity operator, *exh*, that account for a large range of inferences which are traditionally seen as scalar implicatures. In this chapter I present a version of *exh* which deals with singular constituent questions whose short answers are singular and plural NP's. According to my account, the exhaustivity operator makes use of the part-of relation on the domain of singular and plural entities, and of the summing operation over sets of these entities. In chapter 4 I generalize *exh* to deal with domains which use other orderings and maximality operations.

My starting point is the plural example which is repeated in (1) below:

(1) Who came?

Some men came

(2) Groenendijk and Stokhof's exh = $\lambda T \lambda P[T(P) \wedge \neg \exists Q[T(Q) \wedge Q \subset P]]$

The problem with Groenendijk and Stokhof's *exh* (repeated in 2 above) is that the requirement on Q is too strong. We do not want to prevent the existence of proper subsets of comers which contain men, because exh(1) should allow more than one coming man. What we need is to rule out comers who aren't men. My solution (which will be tested in detail shortly) is roughly as follows: We look at every subset Q of plural comers which includes men, and which is closed under summing. The sum of each subset Q, should be a part of the men who come. In this way, we don't exclude the cases where two or more man come, but we make sure that individuals who aren't men don't come – if, for example, a woman comes, one of the relevant subsets, Q, will include her, and therefore, its sum won't be a part of the men who come. Recall that Groenendijk and Stokhof's exhaustivity operator is a relation between the short answer (*some men*, in the case of 1) and the abstract underlying the question (*come*). But in order to capture the new condition, we also need the predicate *men*. This predicate is the predicative interpretation of *some men* (*be some men*). I assume that exhaustivity is a relation between the abstract underlying the question and a pair consisting of the generalized quantifier interpretation and the predicative interpretation of the constituent which serves as the short answer.

(3) Let P and Q be variables ranging over sets of the form *X for some $X \subseteq ATOM$.

We associate with noun phrases two interpretations. NP_{ARG} of type <<e,t>,t> and NP_{PRED} of type <e,t>.

Let T be a variable of type <<e,t>,t>×<e,t> (a variable over pairs of sets of sets and sets).

If $\alpha \in EXP_{<<e,t>,t>\times<e,t>}$ and $[\![\alpha]\!] = <\mathcal{T},\mathcal{P}>$, then $[\![\alpha^1]\!] = \mathcal{T}$ and $[\![\alpha^2]\!] = \mathcal{P}$.

$$exh = \lambda T \lambda P[T^1(P) \wedge \forall Q[[T^1(Q) \wedge Q \subseteq P] \rightarrow \sigma Q \sqsubseteq \sigma(T^2 \cap P)]$$

For most cases discussed here we can assume that $T^2 = BE(T^1)$, where $BE = \lambda T \lambda x[T(\lambda y[y=x])]$ (see Partee 1987), but not for some (see Landman 2004). In those cases I will specify T^2 separately.

The first main conjunct of (3) ensures that the short answer is indeed true. The set which is the interpretation of the abstract underlying the question (P) is a member of the set of sets which is the argument interpretation of the short answer (T^1). The short answer doesn't necessarily give us a full answer, i.e. a list of all individuals in P, but nevertheless it tells us something about them: these individuals are members of T^2, the predicative interpretation of the NP given as the short answer. The second main conjunct of (3) gives us the exhaustiveness effect: the biggest plural individual in $T^2 \cap P$ (the set of individuals who fulfill both the restriction posed by the question and the restriction posed by the short answer) is the biggest plural individual which answers the question. This is done as follows. We look at every subset of P, Q, closed under sum formation, which is a member of T^1 (i.e. every set of individuals closed under sum formation which answers the question partially or fully), the largest element in each of these sets (the sum of each set) is part of the largest element in $T^2 \cap P$.

Let us see now how (3) works for a large set of examples.

(4) Who came?

John

$P = ABS(\text{who came?}) \rightsquigarrow \lambda x{*}COME(x,w_0)$

$T^1 = john \rightsquigarrow \lambda P[P(j)]$

$T^2 = BE(john) = \lambda x[\lambda P[P(j)]\ (\lambda y[y=x])] = \lambda x(\lambda y[y=x](j)) = \lambda x(x=j)$

$$Exh(4) = {*}COME(j,w_0) \wedge \forall Q[[Q(j) \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow$$
$$\sigma Q \sqsubseteq \sigma[\lambda x(x=j) \cap \lambda x{*}COME(x,w_0)]]$$

The first conjunct of exh(4) ensures that $\lambda x{*}COME(x,w_0)$ includes at least John, hence $\sigma[\lambda x(x=j) \cap \lambda x{*}COME(x,w_0)]] = j$, and exh(22) reduces to:

$Exh(4) = {*}COME(j,w_0) \wedge \forall Q[[Q(j) \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow \sigma Q \sqsubseteq j]$

In words: John came, and for every subset of comers (closed under sum formation), which includes John, its sum is part of John.

Exh(4) is true iff only John came. If John didn't come, the first main conjunct of exh(4) is false, hence exh(4) cannot be true. We need to consider two cases: the case where only John came, and the case where someone else came as well.

<u>Case1: Only John came</u>

$[\![\lambda x{*}COME(x,w_0)]\!] = \{John\}$

$[\![Q]\!] = \{John\}$

$[\![\sigma Q]\!] = John$

$[\![\sigma(\lambda x(x=j) \cap \lambda x{*}COME(x,w_0))]\!] = [\![\sigma(\{John\} \cap \{John\})]\!] = John$

Exh(4) requires that John came and that John $\sqsubseteq$ John. Both conditions are fulfilled, hence Exh(4) is true in this case.

<u>Case2: John and Mary came</u>

For simplicity let us consider the case where John and Mary came, and no one else.

$[\![\lambda x{*}COME(x,w_0)]\!]$ = {John, Mary, John$\sqcup$Mary}

$[\![Q_1]\!]$ = {John}; $[\![Q_2]\!]$ = {John, Mary, John$\sqcup$Mary}

$[\![\sigma Q_1]\!]$ = John; $[\![\sigma Q_2]\!]$ = John$\sqcup$Mary

$[\![\sigma(\lambda x(x{=}j)\cap\lambda x{*}COME(x,w_0))]\!] = [\![\sigma[\{John\}\cap \{John, Mary, John\sqcup Mary\})]\!]$ = John

Exh(4) is false in this case, because σQ_2 (= John$\sqcup$Mary) is not a part of John. It is easy to see that exh(4) is false in cases where more people came.

We now look at a plural case.

(5) Who came?
John and Mary

P= ABS(who came?) $\rightsquigarrow \lambda x{*}COME(x,w_0)$

T^1= john and mary $\rightsquigarrow \lambda P[P(j\sqcup m)]$

T^2=BE(john and mary) = $\lambda x[\lambda P[P(j\sqcup m)]\ (\lambda y[y{=}x])] = \lambda x(\lambda y[y{=}x](j\sqcup m)) = \lambda x(x{=}j\sqcup m)$

$$Exh(5) = {}^*COME(j\sqcup m,w_0) \wedge \forall Q[[Q(j\sqcup m) \wedge Q \subseteq \lambda x{}^*COME(x,w_0)] \rightarrow$$

$$\sigma Q \sqsubseteq \sigma[\lambda x(x=j\sqcup m) \cap \lambda x{}^*COME(x,w_0)]]$$

The first main conjunct of exh(5) ensures that $\lambda x{}^*COME(x,w_0)$ includes at least John and Mary, hence $\sigma[\lambda x(x=j\sqcup m) \cap \lambda x{}^*COME(x,w_0)]] = j\sqcup m$, and exh(5) reduces to:

$$Exh(5) = COME(j\sqcup m,w_0) \wedge \forall Q[[Q(j\sqcup m) \wedge Q \subseteq \lambda x{}^*COME(x,w_0)] \rightarrow \sigma Q \sqsubseteq j\sqcup m]$$

In words: John and Mary came and for every subset of comers (closed under sum formation) which includes John and Mary, its sum is part of John and Mary.

Exh(5) means that only John and Mary came. Exh(5)'s first main conjunct requires that John and Mary came. Again, we'll distinguish between two cases: where nobody else came, and where somebody came as well.

<u>Case1: Only John and Mary came</u>

$[\![\lambda x{}^*COME(,w_0)]\!] = \{John, Mary, John\sqcup Mary\}$

$[\![Q]\!] = \{John, Mary, John\sqcup Mary\}$

$[\![\sigma Q]\!] = John\sqcup Mary$

$[\![\sigma[\lambda x(x=j\sqcup m)]\cap\lambda x{}^*COME(x,w_0)]]\!] = [\![\sigma[\{John\sqcup Mary\}\cap\{John, Mary, John\sqcup Mary]]\!] =$ $John\sqcup Mary$

Exh(5) requires that $John \sqcup Mary$ came and that $John \sqcup Mary \sqsubseteq John \sqcup Mary$. Both conditions are fulfilled, hence Exh(5) is true in this case.

<u>Case2: John and Mary and Bill came</u>

For simplicity let us consider the case where John, Mary and Bill came, and no one else.

$[\![\lambda x^*COME(x,w_0)]\!] = \{John, Mary, Bill, John \sqcup Mary, John \sqcup Bill, Mary \sqcup Bill, John \sqcup Mary \sqcup Bill\}$

$[\![Q_1]\!] = \{John, Mary, John \sqcup Mary\}$; $[\![Q_2]\!] = \{John, Mary, Bill, John \sqcup Mary, John \sqcup Bill, Mary \sqcup Bill, John \sqcup Mary \sqcup Bill\ \}$

$[\![\sigma Q_1]\!] = John \sqcup Mary$; $[\![\sigma Q_2]\!] = John \sqcup Mary \sqcup Bill$

$[\![\sigma[\lambda x(x=j \sqcup m)] \cap \lambda x^*COME(x,w_0)]]\!] = [\![\sigma[\{John \sqcup Mary\} \cap \{\{John, Mary, Bill, John \sqcup Mary, John \sqcup Bill, Mary \sqcup Bill, John \sqcup Mary \sqcup Bill]]\!] = John \sqcup Mary$

Exh(5) is false in this case, because σQ_2 ($=John \sqcup Mary \sqcup Bill$) is not a part of $John \sqcup Mary$. It is easy to see that exh(5) is false in cases where more people came.

I assume that *John and Mary* is ambiguous between a Boolean interpretation $\lambda P[P(j) \wedge P(m)]$, and a sum interpretation $\lambda P[P(j \sqcup m)]$. The above analysis used the sum interpretation. The Boolean interpretation gives the wrong result:

(6) Who came?

John and Mary

P= ABS(who came?) $\rightsquigarrow \lambda x{*}COME(x,w_0)$

T^1= john and mary $\rightsquigarrow \lambda P[P(j) \wedge P(m)]$

T^2=BE(john and mary) = $\lambda x[\lambda P[P(j) \wedge P(m)]\ (\lambda y[y=x])] = \lambda x(\lambda y[y=x](j) \wedge \lambda y[y=x](m)) = \lambda x(x=j \wedge x=m)$

Exh(6) = $[{*}COME(j,w_0) \wedge {*}COME(m,w_0)] \wedge \forall Q[[[Q(j) \wedge Q(m)] \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow \sigma Q \sqsubseteq \sigma[[\lambda x(x=j \wedge x=m)] \cap \lambda x{*}COME(x,w_0)]]]$

The set $\lambda x(x=j \wedge x=m)$ is necessarily empty, hence $\sigma[[\lambda x(x=j \wedge x=m)] \cap \lambda x{*}COME(x,w_0)]]$ is necessarily undefined. This means that Exh(6) is false in the state of affairs in which only John and Mary came, because σQ, John$\sqcup$Mary, is not a part of the ‘undefined element’. But of course, this is the wrong prediction.

Is this a problem? I don’t think so. Notice that when we use in *exh* the Boolean interpretation, the semantics will involve by necessity a condition $\alpha \sqsubseteq \beta$, where β is undefined. We can plausibly assume that the essential undefinedness involved just blocks strengthening with the exhaustivity operator. In other words, strengthening the Boolean interpretation with *exh* gets a reading which is essentially **trivial**. But speakers are not likely to strengthen, if the **non**-strengthened meaning is non-trivial, and the strengthened meaning **is**. This means that the exhaustive reading is not available when we use the Boolean interpretation, and hence, in fact, only strengthening with the plural interpretation $\lambda P[P(j \sqcup m)]$, is available.

The next examples involve disjunction.

(7) Who came?

John or Mary

P= ABS(who came?) $\rightsquigarrow \lambda x{*}COME(x,w_0)$

T^1= john or mary $\rightsquigarrow \lambda P[P(j) \vee P(m)]$

T^2=BE(john or mary) = $\lambda x[(x=j) \vee (x=m)]$

Exh(7) = $[{*}COME(j,w_0) \vee {*}COME(m,w_0)] \wedge \forall Q[[[Q(j) \vee Q(m)] \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow \sigma Q \sqsubseteq \sigma[[\lambda x[(x=j) \vee (x=m)] \cap \lambda x{*}COME(x,w_0)]]]$

The first conjunct of exh(7) ensures that $\lambda x{*}COME(x,w_0)$ includes at least John or that it includes at least Mary, hence $\sigma[[\lambda x[(x=j) \vee (x=m)] \cap \lambda x{*}COME(x,w_0)]]$ is only defined if $\lambda x{*}COME(x,w_0) = j$ or if $\lambda x{*}COME(x,w_0) = m$, and exh(7) reduces to:

Exh(7) = $[{*}COME(j,w_0) \vee {*}COME(m,w_0)] \wedge \forall Q[[[Q(j) \vee Q(m)] \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow (\sigma Q \sqsubseteq j \vee \sigma Q \sqsubseteq m)]$

In words: John or Mary came and for every subset of comers (closed under sum formation) which includes John or which includes Mary, its sum is part of John or part of Mary.

Exh(7) means that only John came or only Mary came. Exh(7)'s first main conjunct requires that John or Mary came. Three cases can be distinguished: only John came, only Mary came, John and somebody else came or Mary and somebody else came.

<u>Case1: Only John came</u>

$[\![\lambda x{*}\text{COME}(x,w_0)]\!] = \{\text{John}\}$

$[\![Q]\!] = \{\text{John}\}$

$[\![\sigma Q]\!] = \text{John}$

$[\![\sigma(\lambda x[(x=j)\vee(x=m)] \cap \lambda x{*}\text{COME}(x,w_0))]\!] = [\![\sigma(\{\text{John, Mary}\}\cap\{\text{John}\})]\!] = \text{John}$

Exh(7) requires that John or Mary came and that John $\sqsubseteq$ John. Both conditions are fulfilled, hence Exh(7) is true in this case.

<u>Case2: Only Mary came</u>

$[\![\lambda x{*}\text{COME}(x,w_0)]\!] = \{\text{Mary}\}$

$[\![Q]\!] = \{\text{Mary}\}$

$[\![\sigma Q]\!] = \text{Mary}$

$[\![\sigma(\lambda x[(x=j)\vee(x=m)] \cap \lambda x{*}\text{COME}(x,w_0))]\!] = [\![\sigma(\{\text{John, Mary}\}\cap\{\text{Mary}\})]\!] = \text{Mary}$

Exh(7) requires that John or Mary came and that Mary $\sqsubseteq$ Mary. Both conditions are fulfilled, hence Exh(7) is true in this case.

Case3: John and Mary came

For simplicity let us consider the case where John and Mary came, and no one else.

$[\![\lambda x^*COME(x,w_0)]\!] = \{John, Mary, John \sqcup Mary\}$

$[\![Q_1]\!] = \{John\}; [\![Q_2]\!] = \{Mary\}; [\![Q_3]\!] = \{John, Mary, John \sqcup Mary\}$

$[\![\sigma Q_1]\!] = John; [\![\sigma Q_2]\!] = Mary; [\![\sigma Q_3]\!] = John \sqcup Mary$

$[\![\sigma(\lambda x[(x=j)) \vee (x=m)]) \cap \lambda x^*COME(x,w_0))]\!] = [\![\sigma(\{John, Mary\} \cap \{John, Mary,$

$John \sqcup Mary\})]\!] = [\![\sigma(\{John, Mary\})]\!] = \perp$

Exh(7) is false in this case, because, for example, σQ_1 (= John) is not a part of $\perp$. It is easy to see that exh(7) is false in also in the general cases where John and someone else came, and where Mary and someone else came.

When discussing examples with *or* in the previous chapter (examples 19-21 in section 2.3), we mentioned the possibility of an exhaustive interpretation for *or*, which doesn't have the exclusivity effect (i.e. John or Mary or both came, and no one else). How can we get this reading? Let us look at (8) and (9):

(8) The guest is John or Mary

(9) The guests are John or Mary

All my informants agreed that (8) presupposes that there is only one guest, and asserts that this guest is John or Mary. Not all my informants were happy with (9), but those who accepted it, understood it as conveying that either that there is only one guest who is John or Mary, or that there are two guests, John and Mary:

(10) $\sigma(*GUEST) = j \vee \sigma(*GUEST) = m \vee \sigma(*GUEST) = j\sqcup m$

This fact might pose a problem for plurality theory (how can we compositionally interpret 9 as 10?). However the problem resolves if we assume that *John or Mary* can have the following set interpretation: $T^2 = \lambda x[(x=j)\vee(x=m)\vee(x= j\sqcup m)]$. Using this interpretation, we can get an exhaustivity effect without the exclusivity effect.

(11) Who came?

John or Mary

P= ABS(who came?) $\rightsquigarrow \lambda x{*}COME(x,w_0)$

T^1= john or mary $\rightsquigarrow \lambda P[P(j)\vee P(m)]$

T^2 = john or mary $\rightsquigarrow \lambda x[(x=j)\vee(x=m)\vee(x= j\sqcup m)]$

$Exh(11) = [*COME(j,w_0)\vee *COME(m,w_0] \wedge \forall Q[[[Q(j) \vee Q(m)] \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow \sigma Q \sqsubseteq \sigma[[\lambda x[(x=j)\vee(x=m) \vee(x= j\sqcup m)] \cap \lambda x{*}COME(x,w_0)]]]$

The first conjunct of exh(11) ensures that $\lambda x{*}COME(x,w_0)$ includes John or Mary, hence $\sigma[[\lambda x[(x=j)\vee(x=m)\vee(x= j\sqcup m)] \cap \lambda x{*}COME(x,w_0)]]$ is only defined if $\lambda x{*}COME(x,w_0) = \{j\}$ or if $\lambda x{*}COME(x,w_0) = \{m\}$ or if $\lambda x{*}COME(x,w_0) = \{j,m, j\sqcup m\}$, and exh(10) reduces to:

$Exh(11) = [*COME(j,w_0) \vee *COME(m,w_0)] \wedge \forall Q[[[Q(j) \vee Q(m)] \wedge Q \subseteq \lambda x*COME(x,w_0)] \rightarrow (\sigma Q \sqsubseteq j \vee \sigma Q \sqsubseteq m \vee \sigma Q \sqsubseteq j \sqcup m)]$

In words: John or Mary came and for every subset of comers (closed under sum formation) which includes John or which includes Mary, its sum is part of John or part of Mary or part of John and Mary.

It is not hard to see that exh(11) means that only John came or only Mary came or only John and Mary came.

A similar problem for plurality theory arises with constituents of the form *John and (Mary or Sarah)*. (12) has the interpretation (13):

(12) The guests are John and (Mary or Sarah)

(13) $\sigma(*GUEST) = j \sqcup m \vee \sigma(*GUEST) = j \sqcup s$

Again, for my purposes it is enough to assume that *John and (Mary or Sarah)* has the following set interpretation: $T^2 = \lambda x([x=j \sqcup m] \vee [x= j \sqcup s])$.

(14) Who came?

John and (Mary or Sarah)

P= ABS(who came?) $\rightsquigarrow \lambda x*COME(x,w_0)$

T^1 = john and (mary or sarah) $\rightsquigarrow \lambda P[P(j) \wedge (P(m) \vee P(s))]$

T^2 = john and (mary or sarah) $\rightsquigarrow \lambda x([x=j\sqcup m] \vee [x= j\sqcup s])$

$Exh(14) = [{}^*COME(j,w_0) \wedge ({}^*COME(m,w_0) \;\vee {}^*COME(s,w_0))] \;\wedge \forall Q[[[Q(j) \wedge ((Q(m) \vee Q(s))] \;\wedge Q \subseteq \lambda x{}^*COME(x,w_0)] \rightarrow \sigma Q \sqsubseteq \sigma[\lambda x([x=j\sqcup m] \vee [x= j\sqcup s]) \cap \lambda x{}^*COME(x,w_0)]]]$

The first conjunct of exh(14) ensures that $\lambda x{}^*COME(x,w_0)$ includes John and Mary, or includes John and Sarah, hence $\sigma[\lambda x([x=j\sqcup m] \vee [x= j\sqcup s]) \cap \lambda x{}^*COME(x,w_0)]]$ is only defined if $\lambda x{}^*COME(x,w_0) = \{j\sqcup m\}$ or if $\lambda x{}^*COME(x,w_0) = \{j\sqcup s\}$; and exh(14) reduces to:

$Exh(14) = [{}^*COME(j,w_0) \wedge ({}^*COME(m,w_0) \;\vee {}^*COME(s,w_0))] \;\wedge \forall Q[[[Q(j) \wedge ((Q(m) \vee Q(s))] \;\wedge Q \subseteq \lambda x{}^*COME(x,w_0)] \rightarrow (\sigma Q \sqsubseteq j\sqcup m \vee \sigma Q \sqsubseteq j\sqcup s)]$

In words: John and (Mary or Sarah) came, and for every subset of comers (closed under sum formation), which includes John and (Mary or Sarah), its sum is a part of John and Mary or is a part of John and Sarah.

Exh(14) means that only John and Mary came or only John and Sarah came. We'll distinguish between three cases: where only John and Mary came, where only John and Sarah came and where John, Mary and somebody else came or John, Sarah and somebody else came.

Case1: Only John and Mary came

$[\![\lambda x^*COME(x,w_0)]\!] = \{John, Mary, John\sqcup Mary\}$

$[\![Q]\!] = \{John, Mary, John\sqcup Mary\}$

$[\![\sigma Q]\!] = John\sqcup Mary$

$[\![\sigma[\lambda x([x=j\sqcup m]\vee[x= j\sqcup s])\cap\lambda x^*COME(x,w_0)]]\!] = [\![\sigma[\{John\sqcup Mary, John\sqcup Sarah\}\cap\{John, Mary, John\sqcup Mary\}]]\!] = [\![\sigma[\{John\sqcup Mary\}]]\!] = John\sqcup Mary$

Exh(14) requires that John⊔Mary or John⊔Sarah came and that John⊔Mary ⊑ John⊔Mary. Both conditions are fulfilled, hence Exh(14) is true in this case.

Case2: Only John and Sarah came

$[\![\lambda x^*COME(x,w_0)]\!] = \{John, Sarah, John\sqcup Sarah\}$

$[\![Q]\!] = \{John, Mary, John\sqcup Sarah\}$

$[\![\sigma Q]\!] = John\sqcup Sarah$

$[\![\sigma[\lambda x([x=j\sqcup m]\vee[x= j\sqcup s])\cap\lambda x^*COME(x,w_0)]]\!] = [\![\sigma[\{John\sqcup Mary, John\sqcup Sarah\}\cap\{John, Mary, John\sqcup Sarah\}]]\!] = [\![\sigma[\{John\sqcup Sarah\}]]\!] = John\sqcup Sarah$

Exh(14) requires that John⊔Mary or John⊔Sarah came and that John⊔Sarah ⊑ John⊔Sarah. Both conditions are fulfilled, hence Exh(14) is true in this case.

Case3: John, Mary and Sarah came

For simplicity let us consider the case where John, Mary and Sarah came, and no one else cames.

$[\![\lambda x{*}COME(x,w_0)]\!]$ = {John, Mary, Sarah, John⊔Mary, John⊔Sarah, Sarah⊔Mary, John⊔Mary⊔Sarah}

$[\![Q_1]\!]$ = {John, Mary, John⊔Mary}; $[\![Q_2]\!]$ = {John, Sarah, John⊔Sarah}; $[\![Q_3]\!]$ = {John, Mary, Sarah, John⊔Mary, John⊔Sarah, Sarah⊔Mary, John⊔Mary⊔Sarah}

$[\![\sigma Q_1]\!]$ = John⊔Mary; $[\![\sigma Q_2]\!]$ = John⊔Sarah; $[\![\sigma Q_3]\!]$ = John⊔Mary⊔Sarah

$[\![\sigma[\lambda x([x=j\sqcup m]\vee[x= j\sqcup s])\cap\lambda x{*}COME(x,w_0)]]\!]$ = $[\![\sigma[$\{John⊔Mary, John⊔Sarah\}$\cap$\{John, Mary, Sarah, John⊔Mary, John⊔Sarah, Sarah⊔Mary, John⊔Mary⊔Sarah\}$]]\!]$ = $[\![\sigma[$\{ John⊔Mary, John⊔Sarah\}$]]\!]$ = $\perp$

Exh(14) is false in this case, because σQ_1 (=John⊔Mary) is not a part of $\perp$. It is easy to see that exh(14) is false in also in the general cases where John, Mary and someone else came, and where John, Sarah and someone else came.

Concerning examples (11) and (14), there is a general problem of determining T^2. How do we get the above predicative interpretations of the complex noun phrases? These problems have been discussed to some extent in Winter (1998) and Landman (2004). I do not have a theory to offer on this account – this problem is not the one this thesis is about. However, it is plausible, that any current theory of predicative interpretations should derive the meanings given as available predicate interpretations. What I show is that **given that**, my account of *exh* makes the correct predictions.

Let us consider now some examples with quantifiers and numerals.

(15) Who came?

some man

P= ABS(who came?) $\rightsquigarrow \lambda x{*}COME(x,w_0)$

T^1= some man $\rightsquigarrow \lambda P[\lambda xMAN(x,w_0)\cap P\neq\varnothing]$

T^2= BE(some man) = $\lambda x[\lambda P[\lambda zMAN(z,w_0)\cap P\neq\varnothing](\lambda y[y=x])] = \lambda x[\lambda zMAN(z,w_0)$
$\cap\lambda y[y=x] \neq\varnothing] = \lambda x[\{x\}\cap\lambda zMAN(z,w_0) \neq\varnothing] = \lambda xMAN(x,w_0)$

exh(15) = $\lambda xMAN(x,w_0)\cap\lambda x{*}COME(x,w_0)\neq\varnothing \wedge\forall Q[[\lambda xMAN(x,w_0)\cap Q\neq\varnothing \wedge Q \subseteq$
$\lambda x{*}COME(x,w_0)] \rightarrow \sigma Q \sqsubseteq \sigma(\lambda xMAN(x,w_0)\cap \lambda x{*}COME(x,w_0))]$

In words: Some man came, and for every subset of comers (closed under sum formation) which include singular men, its sum is a part of the man who came.

The first main conjunct of exh(15) requires that there is at least one man who came. $\lambda xMAN(x,w_0)$ is a set of atoms. Thus $\sigma(\lambda xMAN(x,w_0)\cap \lambda x{*}COME(x,w_0))$ is defined only if $\lambda x{*}COME(x,w_0)$ contains only one man. $\lambda x{*}COME(x,w_0)$ cannot contain non-men, if it did, then $\lambda x{*}COME(x,w_0)$ itself would be a subset whose largest member is not a part of the man who came.

Exh(15) means that one man came, and nobody else come. Let us consider 3 cases: only one man came (and nobody else come), only men came, a man and a non-man came.

Case1: One man came, and no one else

$[\![\lambda x MAN(x,w_0)]\!]$ = {John, Harry, Bill}

$[\![\lambda x^*COME(x,w_0)]\!]$ = {John}

$[\![Q]\!]$ = {John}

$[\![\sigma Q]\!]$ = John

$[\![\sigma(\lambda x MAN(x,w_0) \cap \lambda x^*COME(x,w_0))]\!] = [\![\sigma(\{John, Harry, Bill\} \cap \{John\})]\!]$ = John

Exh(15) requires that the set {John, Harry}∩{John} be non empty, and that John $\sqsubseteq$ John. Both conditions are fulfilled, hence Exh(15) is true in this case.

Case2: Only men came

For simplicity let us consider the case where John and Harry came, and no one else.

$[\![\lambda x MAN(x,w_0)]\!]$ = {John, Harry, Bill}

$[\![\lambda x^*COME(x,w_0)]\!]$ = {John, Harry, John⊔Harry}

$[\![Q_1]\!]$ = {John}; $[\![Q_2]\!]$ = {Harry}; $[\![Q_3]\!]$ = {John, Harry, John⊔Harry}

$[\![\sigma Q_1]\!]$ = John; $[\![\sigma Q_2]\!]$ = Harry; $[\![\sigma Q_3]\!]$ = John⊔Harry

$[\![\sigma(\lambda x MAN(x,w_0) \cap \lambda x^*COME(x,w_0))]\!] = [\![\sigma(\{John, Harry, Bill\} \cap \{John, Harry, John \sqcup Harry\})]\!] = [\![\sigma(\{John, Harry\})]\!] = \perp$

Exh(15) is false in this case because σQ_1 (=John), for example, is not a part of $\perp$

<u>Case3: A man and a non-man came</u>

For simplicity let us consider the case where John and Mary came, and no one else.

$[\![\lambda x MAN(x,w_0)]\!]$ = {John, Harry, Bill}

$[\![\lambda x WOMAN(x,w_0)]\!]$ = {Mary, Sarah}

$[\![\lambda x^*COME(x,w_0)]\!]$ = {John, Mary, John$\sqcup$Mary}

$[\![Q_1]\!]$ = {John}; $[\![Q_2]\!]$ = {John, Mary, John$\sqcup$Mary}

$[\![\sigma Q_1]\!]$ = John; $[\![\sigma Q_2]\!]$ = John$\sqcup$Mary

$[\![\sigma(\lambda x MAN(x,w_0)\cap \lambda x^*COME(x,w_0))]\!] = [\![\sigma(\{John, Harry\}\cap \{John, Mary,$

$John\sqcup Mary\})]\!] = [\![\sigma(\{John\})]\!]$ = John

Exh(15) is false in this case because σQ_2 (=John$\sqcup$Mary) is not a part of John.

Now we come to a case that went wrong for Groenendijk and Stokhof.

(16) Who came?

some men

P= ABS(who came?) $\rightsquigarrow \lambda x^*COME(x,w_0)$

T^1= some men $\rightsquigarrow \lambda P[\exists x\in\lambda x^*MAN(x,w_0): |x| \geq 1 \wedge P(x)]$

T^2= BE(some men) = $\lambda z[\lambda P[\exists x{\in}\lambda x{*}MAN(x,w_0)\text{: } |x| \geq 1 \wedge P(x)](\lambda y[y{=}z])]$ =

$\lambda z[\exists x{\in}\lambda x{*}MAN(x,w_0)\text{: } |x| \geq 1 \wedge \lambda y[y{=}z](x)]$ = $\lambda z[\exists x{\in}\lambda x{*}MAN(x,w_0)\text{: } |x| \geq 1 \wedge$

$x{=}z]$ = $\lambda x{*}MAN(x,w_0)$

$$exh(16) = \exists x{\in}\lambda x{*}MAN(x,w_0)\text{: } |x| \geq 1 \wedge {*}COME(x,w_0) \wedge$$

$$\forall Q[[\exists x{\in}\lambda x{*}MAN(x,w_0)\text{: } |x| \geq 1 \wedge Q(x)] \wedge Q{\subseteq}\lambda x{*}COME(x,w_0)] \rightarrow$$

$$\sigma Q \sqsubseteq \sigma(\lambda x{*}MAN(x,w_0) \cap \lambda x{*}COME(x,w_0))]$$

In words: at least one man came, and for every subset of comers (closed under sum) which include men, its sum is a part of the men who came.

Exh(16) means that only men came. Let us consider 2 cases: a case where only men came, and a case where non-men came as well.

<u>Case1: Only men came</u>

For simplicity let us consider the case where John and Harry came, and no one else.

$[\![\lambda xMAN(x,w_0)]\!]$ = {John, Harry}

$[\![\lambda x{*}MAN(x,w_0)]\!]$ = {John, Harry, John$\sqcup$Harry }

$[\![\lambda x{*}COME(x,w_0)]\!]$ = {John, Harry, John$\sqcup$Harry}

$[\![Q_1]\!]$ = {John}; $[\![Q_2]\!]$ = {Harry}; $[\![Q_3]\!]$ = {John, Harry, John$\sqcup$Harry}

$[\![\sigma Q_1]\!]$ = John; $[\![\sigma Q_2]\!]$ = Harry; $[\![\sigma Q_3]\!]$ = John$\sqcup$Harry

$[\![\sigma({*}MAN \cap \lambda x{*}COME(x,w_0))]\!]$ = $[\![\sigma(\{John, Harry, John\sqcup Harry\})]\!]$ = John$\sqcup$Harry

Exh(16) requires that at least one man came, and that John, Harry and John⊔Harry be all part of John⊔Harry. All conditions are fulfilled, hence Exh(16) is true in this case.

Case2: Men and a non-men came

For simplicity let us consider the case where John, Harry and Mary came, and no one else.

$[\![\lambda x MAN(x,w_0)]\!]$ = {John, Harry}

$[\![\lambda x{*}MAN(x,w_0)]\!]$ = {John, Harry, John⊔Harry }

$[\![\lambda x WOMAN(x,w_0)]\!]$ = {Mary, Sarah}

$[\![\lambda x{*}COME(x,w_0)]\!]$ = {John, Mary, Harry, John⊔Mary, John⊔Harry, Mary⊔Harry, John⊔Mary⊔Harry}

$[\![Q_1]\!]$ = {John}; $[\![Q_1]\!]$ = {Harry}; $[\![Q_3]\!]$ = {John, Mary, John⊔Mary}; $[\![Q_4]\!]$ = {John, Harry, John⊔Harry}; $[\![Q_5]\!]$ = {John, Mary, Harry, John⊔Mary, John⊔Harry, Mary⊔Harry, John⊔Mary⊔Harry}

$[\![\sigma Q_1]\!]$ = John; $[\![\sigma Q_2]\!]$ = Harry; $[\![\sigma Q_3]\!]$ = John⊔Mary; $[\![\sigma Q_4]\!]$ = John⊔Harry; $[\![\sigma Q_5]\!]$ = John⊔Mary⊔Harry

$[\![\sigma({*}MAN \cap \lambda x{*}COME(x,w_0))]\!]$ = $[\![\sigma($\{John, Harry, John⊔Harry\}$)]\!]$ = John⊔Harry

Exh(16) is false in this case because σQ_3 (=John⊔Mary), for example, is not a part of John⊔Harry.

Thus the current analysis improves over Groenendijk and Stokhof. We correctly predict that *some men came* as an answer to *Who came?* implicates that only men came, and not that only one man comes. If the question includes a contextual restriction, C (see the discussion in chapter 2, section 2.3), the implicature is weakened to only C's who are men came.

It is important to note here that there are two other implicatures which are traditionally associated with *Some men came* which we do not account for so far:

1. More than one man came.
2. Not all men came

We analyzed *some men* as being synonymous to *at least one man*. And, indeed, *At least one man came* as an answer to *who came?* does not implicate 1 and 2 above. However, *Some men came* **does** implicate that more than one men came. I think that this is a **conventional implicature** associated with the use of the plural noun, *men*. We could, of course, analyze *men* as denoting only non atomic elements in *MAN. But then, the sentence *There were no men at the party* would be compatible with exactly one man being at the party, which is wrong. This point has been made repeatedly in the literature about plurality, see, for example, Landman (2000) and references therein.

Concerning 2, I'm not at all convinced that it is a real implicature of *Some men came* in the context of *Who came?*. Obviously, there are more contexts where *Some men* *came* and *not all men came* are true than contexts where *Some men came* and *all men came* are true, but I don't think that the speaker who answers *who came?* by *Some men* commits herself only to states of affairs where not all men came. However, uttered out of the blue *Some men came* can implicate that not all men come, especially when *some* is focused. We will see in the next chapter that in the context of the question *How many men came?,* exhaustivity predicts that *Some men came* implicates that not all men came (and doesn't implicate that only men came).

Next we discuss examples with numerals.

(17) Who came?

Three men

P= ABS(who came?) $\rightsquigarrow \lambda x{*}COME(x,w_0)$

T^1= three men $\rightsquigarrow \lambda P[\exists x \in \lambda x{*}MAN(x,w_0)\text{: } |x| = 3 \wedge P(x)]$

T^2=BE(three men) = $\lambda z[\lambda P[\exists x \in \lambda x{*}MAN(x,w_0)\text{: } |x| = 3 \wedge P(x)](\lambda y[y=z])] =$

$\lambda z[\exists x \in \lambda x{*}MAN(x,w_0)\text{: } |x| = 3 \wedge \lambda y[y=x](z)] = \lambda z[\exists x \in \lambda x{*}MAN(x,w_0)\text{: } |x| = 3 \wedge$

$x=z] = \lambda x[{*}MAN(x,w_0) \wedge |x|=3]$

exh(17) = $\exists x \in \lambda x{*}MAN(x,w_0)\text{: } |x| = 3 \wedge {*}COME(x,w_0) \wedge$

$\forall Q[[\exists x \in \lambda x{*}MAN(x,w_0)\text{: } |x| = 3 \wedge Q(x)] \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow$

$\sigma Q \sqsubseteq \sigma(\lambda x[{*}MAN(x,w_0) \wedge |x|=3] \cap \lambda x{*}COME(x,w_0))]$

In words: there is a sum of men with exactly 3 atomic elements who came, and for every subset of comers (closed under sum formation) which includes a sum of men with exactly 3 atomic elements, its sum is a part of the men with exactly 3 atomic elements who came.

The first main conjunct of exh(17) ensures that at least 3 men came, hence the set $(\lambda x[*MAN(x,w_0) \wedge |x|=3] \cap \lambda x*COME(x,w_0))$ is non-empty. If there are exactly 3 men that came, this set is a singleton, and $\sigma(\lambda x[*MAN(x,w_0) \wedge |x|=3] \cap \lambda x*COME(x,w_0))]$ is defined.

Exh(17) means that exactly three men came, and no one else. Let us consider 3 cases: a case where exactly 3 men came and no one else, a case where more than 3 men came and no one else, and a case where exactly 3 men and one woman came.

<u>Case1: Only three men came, and no one else</u>

$[\![\lambda x MAN(x,w_0)]\!]$ = {John, Harry, Bill}

$[\![\lambda x*MAN(x,w_0)]\!]$ = {John, Harry, Bill, John⊔Harry, John⊔Bill, Harry⊔Bill, John⊔Harry⊔Bill}

$[\![\lambda x*COME(x,w_0)]\!]$ = {John, Harry, Bill, John⊔Harry, John⊔Bill, Harry⊔Bill, John⊔Harry⊔Bill}

$[\![Q]\!]$ = {John, Harry, Bill, John⊔Harry, John⊔Bill, Harry⊔Bill, John⊔Harry⊔Bill}

$[\![\sigma Q]\!]$ = John⊔Harry⊔Bill

$[\![\sigma(\lambda x[{}^*MAN(x,w_0) \wedge |x|=3] \cap \lambda x{}^*COME(x,w_0))]\!] = [\![\sigma(\{John \sqcup Harry \sqcup Bill\})]\!] =$

$John \sqcup Harry \sqcup Bill$

Exh(17) requires that there be a sum of men with exactly 3 elements that came, and that John⊔Harry⊔Bill be a part John⊔Harry⊔Bill. These conditions are fulfilled, hence Exh(17) is true in this case.

<u>Case2: More than three men came, and no one else</u>

$[\![\lambda xMAN(x,w_0)]\!]$ = {John, Harry, Bill, Fred}

$[\![\lambda x{}^*MAN(x,w_0)]\!]$ = {John, Harry, Bill, Fred, John⊔Harry, John⊔Bill, John⊔Fred, Harry⊔Bill, Harry⊔Fred, Bill⊔Fred, John⊔Harry⊔Bill, John⊔Harry⊔Fred, John⊔Bill⊔Fred Harry⊔Bill⊔Fred, John⊔Harry⊔Bill⊔Fred}

$[\![\lambda x{}^*COME(x,w_0)]\!]$ = {John, Harry, Bill, Fred, John⊔Harry, John⊔Bill, John⊔Fred, Harry⊔Bill, Harry⊔Fred, Bill⊔Fred, John⊔Harry⊔Bill, John⊔Harry⊔Fred, John⊔Bill⊔Fred, Harry⊔Bill⊔Fred, John⊔Harry⊔Bill⊔Fred}

$[\![Q_1]\!]$ = {John, Harry, Bill, John⊔Harry, John⊔Bill, Harry⊔Bill, John⊔Harry⊔Bill}

$[\![Q_2]\!]$ = {John, Harry, Fred, John⊔Harry, John⊔Fred, Harry⊔Fred, John⊔Harry⊔Fred}

$[\![Q_3]\!]$ = {John, Bill, Fred, John⊔Bill, John⊔Fred, Bill⊔Fred, John⊔Bill⊔Fred}

$[\![Q_4]\!]$ = {John, Harry, Bill, Fred, John⊔Harry, John⊔Bill, John⊔Fred, Harry⊔Bill, Harry⊔Fred, Bill⊔Fred, John⊔Harry⊔Bill, John⊔Harry⊔Fred, John⊔Bill⊔Fred Harry⊔Bill⊔Fred, John⊔Harry⊔Bill⊔Fred}

$[\![\sigma Q_1]\!]$ = John⊔Harry⊔Bill; $[\![\sigma Q_2]\!]$ = John⊔Harry⊔Fred; $[\![\sigma Q_3]\!]$ = John⊔Bill⊔Fred;

$[\![\sigma Q_4]\!]$ = John⊔Harry⊔Bill⊔Fred

$[\![\sigma(\lambda x[*MAN(x,w_0) \wedge |x|=3] \cap \lambda x*COME(x,w_0))]\!] = [\![\sigma(\{$ John⊔Harry⊔Bill, John⊔Harry⊔Fred, John⊔Bill⊔Fred$)]\!] = \bot$

In this case $\sigma(\lambda x[*MAN(x,w_0) \wedge |x|=3] \cap \lambda x*COME(x,w_0))$ is undefined. For all Q's, σQ is not part of ⊥, hence exh(34) is false.

<u>Case3: Exactly three men and one woman came, and no one else</u>

$[\![\lambda xMAN(x,w_0)]\!]$ = {John, Harry, Bill, Fred}

$[\![\lambda x*MAN(x,w_0)]\!]$ = {John, Harry, Bill, Fred, John⊔Harry, John⊔Bill, John⊔Fred, Harry⊔Bill, Harry⊔Fred, Bill⊔Fred, John⊔Harry⊔Bill, John⊔Harry⊔Fred, John⊔Bill⊔Fred Harry⊔Bill⊔Fred, John⊔Harry⊔Bill⊔Fred}

$[\![\lambda xWOMAN(x,w_0)]\!]$ = {Mary, Sarah}

$[\![\lambda x*COME(x,w_0)]\!]$ = {John, Harry, Bill, Sarah, John⊔Harry, John⊔Bill, John⊔Sarah, Harry⊔Bill, Harry⊔Sarah, Bill⊔Sarah, John⊔Harry⊔Bill, John⊔Harry⊔Sarah, John⊔Bill⊔Sarah, Harry⊔Bill⊔Sarah, John⊔Harry⊔Bill⊔Sarah}

$[\![Q_1]\!]$ = {John, Harry, Bill, John⊔Harry, John⊔Bill, John⊔Harry⊔Bill}

$[\![Q_2]\!]$ = {John, Harry, Bill, Sarah, John⊔Harry, John⊔Bill, John⊔Sarah, Harry⊔Bill, Harry⊔Sarah, Bill⊔Sarah, John⊔Harry⊔Bill, John⊔Harry⊔Sarah, John⊔Bill⊔Sarah, Harry⊔Bill⊔Sarah, John⊔Harry⊔Bill⊔Sarah}

$[\![\sigma Q_1]\!]$ = John⊔Harry⊔Bill; $[\![\sigma Q_2]\!]$ = John⊔Harry⊔Bill⊔Sarah

$[\![\sigma(\lambda x[*MAN(x,w_0) \wedge |x|=3] \cap \lambda x*COME(x,w_0))]\!] = [\![\sigma(\{John \sqcup Harry \sqcup Bill\})]\!] =$ John⊔Harry⊔Bill

Exh(17) is false because, σQ_2 (John⊔Harry⊔Bill⊔Sarah) is a not part of John⊔Harry⊔Bill.

The effect of exhaustivization in this example is twofold: exactly three men came, and only men came. According to definition (18) (in section 2.2, chapter 2), both are implicatures of (17). This seems right for this context. However, out of the blue *Three men came* can have the first implicature without the second. We will see in the next chapter that exhaustivity predicts this fact when *Three men came* is considered as an answer to *How many men came*?.

(18) Who came?

At least three men

P= ABS(who came?) $\rightsquigarrow \lambda x*COME(x,w_0)$

T^1= at least three men $\rightsquigarrow \lambda P[\exists x \in \lambda x*MAN(x,w_0): |x| \geq 3 \wedge P(x)]$

T^2 = BE(at least three men) = $\lambda z[\lambda P[\exists x \in \lambda x*MAN(x,w_0): |x| \geq 3 \wedge P(x)](\lambda y[y=z])] = \lambda z[\exists x \in \lambda x*MAN(x,w_0): |x| \geq 3 \wedge \lambda y[y=x](z)] = \lambda z[\exists x \in \lambda x*MAN(x,w_0): |x| \geq 3 \wedge x=z] = \lambda x[*MAN(x,w_0) \wedge |x| \geq 3]$

$$\text{exh}(18) = \exists x{\in}\lambda x{*}\text{MAN}(x,w_0){:}\ |x| \geq 3 \wedge {*}\text{COME}(x,w_0) \wedge$$

$$\forall Q[[\exists x{\in}\lambda x{*}\text{MAN}(x,w_0){:}\ |x| \geq 3 \wedge Q(x)] \wedge Q \subseteq \lambda x{*}\text{COME}(x,w_0)] \rightarrow$$

$$\sigma Q \sqsubseteq \sigma(\lambda x[\lambda x{*}\text{MAN}(x,w_0) \wedge |x| \geq 3] \cap \lambda x{*}\text{COME}(x,w_0))]$$

In words: there is a sum of men with at least 3 atomic elements who came, and for every subset of comers (closed under sum formation) which includes a sum of men with at least 3 atomic elements, its sum is a part of the men with at least 3 atomic elements who came.

Exh(18) means that at least three men came, and no one else. Let us consider 2 cases: a case where 4 men came and no one else, and a case where 3 men and one woman came.

<u>Case1: four men came, and no one else</u>

$[\![\lambda x\text{MAN}(x,w_0)]\!]$ = {John, Harry, Bill, Fred}

$[\![\lambda x{*}\text{MAN}(x,w_0)]\!]$ = {John, Harry, Bill, Fred, John⊔Harry, John⊔Bill, John⊔Fred, Harry⊔Bill, Harry⊔Fred, Bill⊔Fred, John⊔Harry⊔Bill, John⊔Harry⊔Fred, John⊔Bill⊔Fred Harry⊔Bill⊔Fred, John⊔Harry⊔Bill⊔Fred}

$[\![\lambda x{*}\text{COME}(x,w_0)]\!]$ = {John, Harry, Bill, Fred, John⊔Harry, John⊔Bill, John⊔Fred, Harry⊔Bill, Harry⊔Fred, Bill⊔Fred, John⊔Harry⊔Bill, John⊔Harry⊔Fred, John⊔Bill⊔Fred, Harry⊔Bill⊔Fred, John⊔Harry⊔Bill⊔Fred}

$[\![Q_1]\!]$ = {John, Harry, Bill, John⊔Harry, John⊔Bill, John⊔Harry⊔Bill}

$[\![Q_2]\!]$ = {John, Harry, Fred, John⊔Harry, John⊔Fred, John⊔Harry⊔Fred}

$\llbracket Q_3 \rrbracket$ = {John, Bill, Fred, John$\sqcup$Bill, John$\sqcup$Frd, John$\sqcup$Bill$\sqcup$Fred}

$\llbracket Q_4 \rrbracket$ = {John, Harry, Bill, Fred, John$\sqcup$Harry, John$\sqcup$Bill, John$\sqcup$Fred, Harry$\sqcup$Bill, Harry$\sqcup$Fred, Bill$\sqcup$Fred, John$\sqcup$Harry$\sqcup$Bill, John$\sqcup$Harry$\sqcup$Fred, John$\sqcup$Bill$\sqcup$Fred Harry$\sqcup$Bill$\sqcup$Fred, John$\sqcup$Harry$\sqcup$Bill$\sqcup$Fred}

$\llbracket \sigma Q_1 \rrbracket$ = John$\sqcup$Harry$\sqcup$Bill; $\llbracket \sigma Q_2 \rrbracket$ = John$\sqcup$Harry$\sqcup$Fred; $\llbracket \sigma Q_3 \rrbracket$ = John$\sqcup$Bill$\sqcup$Fred;

$\llbracket \sigma Q_4 \rrbracket$ = John$\sqcup$Harry$\sqcup$Bill$\sqcup$Fred

$\llbracket \sigma(\lambda x[\lambda x^*MAN(x,w_0) \wedge |x| \geq 3] \cap \lambda x^*COME(x,w_0)) \rrbracket = \llbracket \sigma($\{ John$\sqcup$Harry$\sqcup$Bill, John$\sqcup$Harry$\sqcup$Fred, John$\sqcup$Bill$\sqcup$Fred, John$\sqcup$Harry$\sqcup$Bill$\sqcup$Fred\}$) \rrbracket$ = John$\sqcup$Harry$\sqcup$Bill$\sqcup$Fred

Exh(18) requires that there be a plural man with at least 3 elements that came, and that John$\sqcup$Harry$\sqcup$Bill, John$\sqcup$Harry$\sqcup$Fred, John$\sqcup$Bill$\sqcup$Fred, John$\sqcup$Harry$\sqcup$Bill$\sqcup$Fred be all parts of John$\sqcup$Harry$\sqcup$Bill$\sqcup$Fred. All these conditions are fulfilled, hence Exh(18) is true in this case.

<u>Case2: Exactly three men and one woman came, and no one else</u>

$\llbracket \lambda x MAN(x,w_0) \rrbracket$ = {John, Harry, Bill, Fred}

$\llbracket \lambda x^*MAN(x,w_0) \rrbracket$ = {John, Harry, Bill, Fred, John$\sqcup$Harry, John$\sqcup$Bill, John$\sqcup$Fred, Harry$\sqcup$Bill, Harry$\sqcup$Fred, Bill$\sqcup$Fred, John$\sqcup$Harry$\sqcup$Bill, John$\sqcup$Harry$\sqcup$Fred, John$\sqcup$Bill$\sqcup$Fred Harry$\sqcup$Bill$\sqcup$Fred, John$\sqcup$Harry$\sqcup$Bill$\sqcup$Fred}

$\llbracket \lambda x WOMAN(x,w_0) \rrbracket$ = {Mary, Sarah}

$[\![\lambda x^*COME(x,w_0)]\!]$ = {John, Harry, Bill, Sarah, John⊔Harry, John⊔Bill, John⊔Sarah, Harry⊔Bill, Harry⊔Sarah, Bill⊔Sarah, John⊔Harry⊔Bill, John⊔Harry⊔Sarah, John⊔Bill⊔Sarah, Harry⊔Bill⊔Sarah, John⊔Harry⊔Bill⊔Sarah}

$[\![Q_1]\!]$ = {John, Harry, Bill, John⊔Harry, John⊔Bill, John⊔Harry⊔Bill}

$[\![Q_2]\!]$ = {John, Harry, Bill, Sarah, John⊔Harry, John⊔Bill, John⊔Sarah, Harry⊔Bill, Harry⊔Sarah, Bill⊔Sarah, John⊔Harry⊔Bill, John⊔Harry⊔Sarah, John⊔Bill⊔Sarah, Harry⊔Bill⊔Sarah, John⊔Harry⊔Bill⊔Sarah}

$[\![\sigma Q_1]\!]$ = John⊔Harry⊔Bill; $[\![\sigma Q_2]\!]$ = John⊔Harry⊔Bill⊔Sarah

$[\![\sigma(\lambda x[\lambda x^*MAN(x,w_0)\wedge |x| \geq 3]\cap \lambda x^*COME(x,w_0))]\!] = [\![\sigma(\{John \sqcup Harry \sqcup Bill\})]\!]$ = John⊔Harry⊔Bill

Exh(18) is false because, σQ_2 (John⊔Harry⊔Bill⊔Sarah) is a not part of John⊔Harry⊔Bill.

We see that exhaustivity predicts correctly that *At least 3 men came* as an answer to *who came?* does not implicate that exactly 3 men came, but does implicate that only men came.

Next we look at cases with *no* and *all*.

(19) Who came?

No man

P= ABS(who came?) $\rightsquigarrow \lambda x{*}COME(x,w_0)$

T^1= no man $\rightsquigarrow \lambda P[\lambda xMAN(x,w_0) \cap P=\varnothing]$

T^2 = BE(no man) = $\lambda x[\lambda P[\lambda zMAN(z,w_0) \cap P=\varnothing](\lambda y[y=x])]$ =

$\lambda x[\lambda zMAN(z,w_0) \cap (\lambda y[y=x])=\varnothing] = \lambda x[\lambda zMAN(z,w_0) \cap \{x\}=\varnothing]$ = D-$\lambda xMAN(x,w_0)$

exh(19) = $[\lambda xMAN(x,w_0) \cap \lambda x{*}COME(x,w_0)=\varnothing] \wedge$

$\forall Q[[\lambda xMAN(x,w_0) \cap Q=\varnothing] \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow$

$\sigma Q \sqsubseteq \sigma[(\text{D-}\lambda xMAN(x,w_0)) \cap \lambda x{*}COME(x,w_0))]]$

In words: no man came, and for every subset of comers (closed under sum formation) which don't include singular men, its sum is a part of the comers which are not singular men.

Exh(19) simply means that no man came. $\lambda xMAN(x,w_0) \cap \lambda x{*}COME(x,w_0)$ is empty, hence (D-$\lambda xMAN(x,w_0)) \cap \lambda x{*}COME(x,w_0) = \lambda x{*}COME(x,w_0)$. The subsets Q, range over sets of the form *X for X$\subseteq$ATOM, hence, the sum of every such subset of comers Q, is a part of the sum of $\lambda x{*}COME(x,w_0)$. This is the correct prediction. The answer *no man* implicates nothing about who didn't come.

(20) Who came?

No men

$P= ABS(\text{who came?}) \rightsquigarrow \lambda x{*}COME(x,w_0)$

$T^1= \text{ no men} \rightsquigarrow \lambda P[\lambda x{*}MAN(x,w_0) \cap P=\varnothing]$

$T^2 = BE(\text{no men}) = \lambda x[\lambda P[\lambda z{*}MAN(z,w_0) \cap P=\varnothing](\lambda y[y=x])] =$

$\lambda x[\lambda z{*}MAN(z,w_0) \cap (\lambda y[y=x])=\varnothing] = \lambda x[\lambda z{*}MAN(z,w_0) \cap \{x\}=\varnothing] =$

$D\text{-}\lambda x{*}MAN(x,w_0)$

$$exh(20) = [\lambda x{*}MAN(x,w_0) \cap \lambda x{*}COME(x,w_0) = \varnothing] \wedge$$
$$\forall Q[[\lambda x{*}MAN(x,w_0) \cap Q=\varnothing] \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow$$
$$\sigma Q \sqsubseteq \sigma[(D\text{-}\lambda x{*}MAN(x,w_0)) \cap \lambda x{*}COME(x,w_0))]]$$

In words: no men came, and for every subset of comers (closed under sum formation) which don't include men, its sum is a part of the comers which are not men.

Exh(20) too means that no man came. $\lambda x{*}MAN(x,w_0) \cap \lambda x{*}COME(x,w_0)$ is empty, hence $(D\text{-}\lambda x{*}MAN(x,w_0)) \cap \lambda x{*}COME(x,w_0) = \lambda x{*}COME(x,w_0)$. The subsets Q range over sets of the form *X for $X \subseteq ATOM$, hence, the sum of every such subset of comers Q, is a part of the sum of $\lambda x{*}COME(x,w_0)$. This is the correct prediction. The answer *no men* implicates nothing about who didn't come.

(21) Who came?

all men

$P = ABS(\text{who came?}) \rightsquigarrow \lambda x{*}COME(x,w_0)$

$T^1 = \text{all men} \rightsquigarrow \lambda P P(\sqcup(\lambda x{*}MAN(x,w_0)))$

$T^2 = BE(\text{all men}) = \lambda x[\lambda P[P(\sqcup(\lambda z{*}MAN(z,w_0)))](\lambda y[y=x])] =$

$\lambda x[\lambda y[y=x]](\sqcup(\lambda x{*}MAN(x,w_0))) = \lambda x[x=\sqcup(\lambda z{*}MAN(z,w_0))] = \{\sqcup(\lambda x{*}MAN(x,w_0))\}$

$$exh(21) = \lambda x{*}COME(x,w_0)\,(\sqcup(\lambda x{*}MAN(x,w_0))) \wedge$$

$$\forall Q[[Q(\sqcup(\lambda x{*}MAN(x,w_0))) \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow$$

$$\sigma Q \sqsubseteq \sigma(\{\sqcup(\lambda x{*}MAN(x,w_0))\} \cap \lambda x{*}COME(x,w_0))]$$

In words: all men came, and for every subset of comers (closed under sum formation) which includes all men, it sum is a part of the men that came.

Exh(21) means that all men came, and no one else. If, besides all men, Mary, a woman, came as well, $\lambda x{*}COME(x,w_0)$ would be such a Q that falsifies exh(21)'s truth conditions. $\sigma[\lambda x{*}COME(x,w_0)]$, which would have Mary as one of its parts, would not be a part of all the man that came.

Similarly to the case with *John and Bill*, we get the correct exhaustive interpretation of *all men came* with the sum interpretation of *all men*. As we'll see shortly (when we discuss the case of *every man* in example 23 below), the Boolean interpretation $\lambda P[\lambda x MAN(x,w_0) \subseteq P]$ will yield the wrong result (and as before, I will assume that the undefinedness involved will just make strengthening the Boolean interpretation with *exh* unavailable).

Let us consider now a case of a downward entailing NP with a numeral.

(22) Who came?

At most two men

P= ABS(who came?) $\leadsto \lambda x{*}COME(x,w_0)$

T^1= (at most two men)$_{ARG}$ $\leadsto \lambda P[|\sqcup(\lambda x{*}MAN(x,w_0)\cap P)| \leq 2]$

T^2 = (at most two men)$_{PRED}$ $\leadsto \lambda x[{*}MAN(x) \wedge |x| \leq 2]$

Landman (2004) argues that for downward entailing noun phrases such as *at most 2 men*, Partee's BE cannot derive the predicate interpretation from the argument interpretation. My interest here is not to give a theory of predicate interpretations. So, I will just follow Landman and assume that the predicate interpretation of *at most two men* is $\lambda x[{*}MAN(x) \wedge |x| \leq 2]$. This, then is a case where we don't use BE to derive the second element of the pair, and it is the main reason that we used a variable on pairs in our formulation of the exhaustivity operator.

$exh(22) = [|\sqcup(\lambda x{*}MAN(x,w_0) \cap \lambda x{*}COME(x,w_0))| \leq 2] \wedge$

$\forall Q[[[|\sqcup(\lambda x{*}MAN(x,w_0) \cap Q)| \leq 2] \wedge Q \subseteq \lambda x{*}COME(x,w_0)] \rightarrow$

$\sigma Q \sqsubseteq \sigma\ ([\lambda x[{*}MAN(x) \wedge |x| \leq 2] \cap \lambda x{*}COME(x,w_0))]$

In words: the sum of the men that came has at most 2 atomic elements (i.e. at most 2 men came), and for every subset of comers (closed under sum formation) which include at most 2 men, its sum is a part of the comers who are men that have at most 2 atomic elements.

Exh(22) means that no more than two men came, and no one else. To convince ourselves, let us examine 4 cases: no one came, two men came, and no one else, two men and one woman came and no one else, and one woman came and no one else.

<u>Case1: No one came</u>

$[\![\lambda x MAN(x,w_0)]\!] = \{John, Harry\}$

$[\![\lambda x{*}MAN(x,w_0)]\!] = \{John, Harry, John \sqcup Harry\}$

$[\![\lambda x{*}COME(x,w_0)]\!] = \varnothing$

$[\![Q]\!] = \varnothing$

$[\![\sigma Q]\!] = 0$

$[\![\sigma([\lambda x[{*}MAN(x) \wedge |x| \leq 2] \cap \lambda x{*}COME(x,w_0))]]\!] = [\![\sigma\varnothing]\!] = 0$

Exh (22) comes out true in this case. Its first main conjunct is fulfilled, because $|\sigma(\lambda x{*}MAN(x,w_0) \cap \lambda x{*}COME(x,w_0))| = |\sigma(\varnothing)| = 0 \leq 2$. The second conjunct is also fulfilled because $0 \sqsubseteq 0$.

Case2: Two men came, and no one else

$[\![\lambda xMAN(x,w_0)]\!] = \{John, Harry\}$

$[\![\lambda x{*}MAN(x,w_0)]\!] = \{John, Harry, John \sqcup Harry\}$

$[\![\lambda x{*}COME(x,w_0)]\!] = \{John, Harry, John \sqcup Harry\}$

$[\![Q_1]\!] = \varnothing$; $[\![Q_2]\!] = \{John\}$; $[\![Q_3]\!] = \{Harry\}$; $[\![Q_4]\!] = \{John, Harry, John \sqcup Harry\}$

$[\![\sigma Q_1]\!] = 0$; $[\![\sigma Q_2]\!] = John$; $[\![\sigma Q_3]\!] = Harry$; $[\![\sigma Q_4]\!] = John \sqcup Harry$

$[\![\sigma([\lambda x[{*}MAN(x) \wedge |x| \leq 2] \cap \lambda x{*}COME(x,w_0))]]\!] = [\![\sigma([\{John, Harry, John \sqcup Harry\}] \cap \{John, Harry, John \sqcup Harry]\!] = [\![\sigma(\{John, Harry, John \sqcup Harry\})]\!] = John \sqcup Harry$

Exh (22) comes out true in this case. Its first main conjunct is fulfilled, because $|\sigma(\lambda x{*}MAN(x,w_0) \cap \lambda x{*}COME(x,w_0))| = |John \sqcup Harry| = 2 \leq 2$. The second conjunct is also fulfilled because 0, John, Harry and John⊔Harry are all parts of John⊔Harry.

Case3: Two men and one woman came, and no one else

$[\![\lambda xMAN(x,w_0)]\!] = \{John, Harry\}$

$[\![\lambda x{*}MAN(x,w_0)]\!] = \{John, Harry, John \sqcup Harry\}$

$[\![\lambda xWOMAN(x,w_0)]\!] = \{Mary\}$

$[\![\lambda x{*}WOMAN(x,w_0)]\!] = \{Mary\}$

$[\![\lambda x{*}COME(x,w_0)]\!] = \{John, Harry, Mary, John \sqcup Harry, John \sqcup Mary, Mary \sqcup Harry,$

$John \sqcup Harry \sqcup Mary\}$

$[\![Q_1]\!] = \emptyset$; $[\![Q_2]\!] = \{John\}$; $[\![Q_3]\!] = \{Harry\}$; $[\![Q_4]\!] = \{Mary\}$; $[\![Q_5]\!] = \{John, Harry,$

$John \sqcup Harry\}$; $[\![Q_6]\!] = \{John, Mary, John \sqcup Mary\}$; $[\![Q_7]\!] = \{Harry, Mary, Harry \sqcup Mary\}$;

$[\![Q_8]\!] = \{John, Harry, Mary, John \sqcup Harry, John \sqcup Mary, Mary \sqcup Harry,$

$John \sqcup Harry \sqcup Mary\}$

$[\![\sigma Q_1]\!] = 0$; $[\![\sigma Q_2]\!] = John$; $[\![\sigma Q_3]\!] = Harry$; $[\![\sigma Q_4]\!] = Mary$; $[\![\sigma Q_5]\!] = John \sqcup Harry$;

$[\![\sigma Q_6]\!] = John \sqcup Mary$; $[\![\sigma Q_7]\!] = Mary \sqcup Harry$; $[\![\sigma Q_8]\!] = John \sqcup Harry \sqcup Mary$

$[\![\sigma\ ([\lambda x[{*}MAN(x) \wedge |x| \leq 2] \cap \lambda x{*}COME(x,w_0))]]\!]\ =\ [\![\sigma([John, Harry,$

$John \sqcup Harry\}] \cap \{John, Harry, Mary, John \sqcup Harry, John \sqcup Mary, Mary \sqcup Harry,$

$John \sqcup Harry \sqcup Mary\}]\!] = [\![\sigma(\{John, Harry, John \sqcup Harry\})]\!] = John \sqcup Harry$

As desired, Exh (22) comes out as false in this case, because, for example, σQ_8 ($John \sqcup Harry \sqcup Mary$) is not a part of $John \sqcup Harry$.

<u>Case4: one woman came, and no one else</u>

$[\![ATOM]\!] = \{John, Harry, Mary\}$

$[\![\lambda xMAN(x,w_0)]\!] = \{John, Harry\}$

$[\![\lambda x{*}MAN(x,w_0)]\!] = \{John, Harry, John \sqcup Harry\}$

$[\![\lambda xWOMAN(x,w_0)]\!] = \{Mary\}$

$[\![\lambda x^*WOMAN(x,w_0)]\!] = \{Mary\}$

$[\![\lambda x^*COME(x,w_0)]\!] = \{ Mary\}$

$[\![Q_1]\!] = \varnothing$; $[\![Q_2]\!] = \{Mary\}$

$[\![\sigma Q_1]\!] = 0$; $[\![\sigma Q_2]\!] = Mary$

$[\![\sigma([\lambda x[^*MAN(x) \wedge |x| \leq 2] \cap \lambda x^*COME(x,w_0))]]\!] = [\![\sigma([John,$

$Harry, John \sqcup Harry\}]\cap\{Mary]\!] = [\![\sigma\varnothing]\!] = 0$

As desired, Exh (22) comes out as false in this case too. Its first main conjunct is fulfilled, because, σQ_2 (Mary) is not a part of 0.

The last case we look at in this chapter is a case with *every*.

(23) Who came?

Every man

$P= ABS(who\ came?) \rightsquigarrow \lambda x^*COME(x,w_0)$

$T^1= every\ man \rightsquigarrow \lambda P[\lambda xMAN(x,w_0)\subseteq P]$

$T^2= BE(every\ man) = \lambda x[\lambda P[\lambda zMAN(z,w_0)\subseteq P]\ (\lambda y[y=x])] =$

$\lambda x[\lambda zMAN(z,w_0)\subseteq\lambda y[y=x]] = \lambda x[\lambda zMAN(z,w_0)\subseteq\{x\}] = \lambda xMAN(x,w_0)$, if $\lambda xMAN(x,w_0)$ is a singleton set; undefined otherwise

The BE operator does not give us the set we need. As a matter of fact, *every man* does not normally function as a predicate. For example, (24) is infelicitous.

(24) #Nirit is every semantics professor at the party.

However, Landman (2003) shows, that *everyNPs* do sometimes have a predicate interpretation. Consider (25):

(25) The press is every person who writes about the news.

Landman assumes that an *everyNP* can sometimes shift to a collective interpretation (*everyNP* $\rightarrow$ *allNP*), and that the collective shifted interpretations might occur as predicates. I assume that in our case as well, there is some rescue mechanism that shifts the meaning of *every man* to the meaning of *all men*, and hence example (23) is analyzed in the same way as example (21).

Chapter 4

Exhaustivity on Other Domains

In this chapter I generalize the analysis given in the previous chapter so it will apply to other domains besides the domain of singular and plural entities. The plural part-of relation and the sigma operator referred to in *exh* will be generalized to other orderings and maximality operators.

4.1 Exhaustivity on ordered sets of atoms

The formulation of *exh* given in the previous chapter (and which is repeated in 1 below) is especially tailored to sets of pluralities - as it is, it is not applicable to examples such as (2):

(1) Let P and Q be variables ranging over sets of the form *X for some $X \subseteq ATOM$.

We associate with noun phrases two interpretations. NP_{ARG} of type <<e,t>,t> and NP_{PRED} of type <e,t>.

Let T be a variable of type <<e,t>,t>×<e,t> (a variable over pairs of sets of sets and sets).

If $\alpha \in EXP_{<<e,t>,t>\times<e,t>}$ and $[\![\alpha]\!] = <\mathcal{T},\mathcal{P}>$, then $[\![\alpha^1]\!] = \mathcal{T}$ and $[\![\alpha^2]\!] = \mathcal{P}$.

$exh = \lambda T \lambda P[T^1(P) \wedge \forall Q[[T^1(Q) \wedge Q \subseteq P] \rightarrow \sigma Q \sqsubseteq \sigma(T^2 \cap P)]$

(2) How many chairs does John have?

John has $[3]_F$ chairs.

The set of numbers of chairs such that John has that many chairs, is not a set of plural entities closed under the sum operation, but it shares some important properties with such sets. It is not merely a set of atoms, but an **ordered** set of atoms, and it always has a largest element (the exact number of chairs that John has). We can define a **maximum** operation, *max*, on sets of numbers. The maximum of two numbers is the larger of the two. Hence, any finite set of natural numbers is closed under max - the larger numbers of any pair of numbers in some finite set of numbers - is always in the set. We can think of the abstract of a single constituent question, and of the abstract of a *how many?* question as a **join semilattice**.

(3) Let A be a set, let $\leq$ be a partial order, and let *max* be a two place operation s.t. For any a, $b \in A$, max(a,b) is the smallest element s.t. $a \leq max(a,b)$ and $b \leq max(a,b)$.

A structure $<A, \leq>$ is a join semilattice iff for any a, $b \in A$, $max(a,b) \in A$

(4) Let A be a set, let $\leq$ be a partial order, and let $B \subseteq A$, maxB is the unique element in B, if there is such a unique element, s.t. for every $b \in B$, $b \leq maxB$, undefined otherwise.

A structure of the form <*P, $\sqsubseteq$ >, where P$\subseteq$D, and $\sqsubseteq$ is the plural part of relation - is a join semilattice. We defined, $\sqcup$, the sum operation using the partial order $\sqsubseteq$, and we defined *P as closure under $\sqcup$. Note that if P$\neq\emptyset$, σ(*P) is always defined, because *P is closed under sum. A structure of the form <N, ≤ >, where N is a finite set of natural numbers, and ≤ is the smaller or equal relation between numbers, is also a join semilattice. For any two numbers m, n, max(m,n) is the larger of the two, and it is always in N, hence maxN is always defined.

Let us generalize the formulation of exhaustivity given in (1), in such a way that the part-of relation and the sigma operator referred to in (1) will come out just as instantiations of a more general partial order, and a maximum operator.

(5) Let P, Q be variables ranging over sets, partially ordered by ≤., s.t. <Q, ≤ > is a join semilattice.

We associate with NPs or numerals, two interpretations: an interpretation of type <<e,t>,t> and an interpretation of type <e,t>.

Let T be a variable of type <<e,t>,t>×<e,t> (a variable over pairs of sets of sets and sets). If $\alpha \in EXP_{<<e,t>,t>\times<e,t>}$ and $[\![\alpha]\!] = <\mathcal{T},\mathcal{P}>$, then $[\![\alpha^1]\!] = \mathcal{T}$ and $[\![\alpha^2]\!] = \mathcal{P}$.

$$exh = \lambda T\lambda P[T^1(P)\wedge\forall Q[[T^1(Q)\wedge Q \subseteq P] \rightarrow maxQ \leq\ max(T^2\cap P)]$$

Note that in the definition of *exh*, we do not require <P, ≤ > to be a join semilattice, but we look only at subsets of P, Q such that <Q, ≤ > are join semilattices. In the cases of plural NP's, and numerals, <P, ≤ > itself is a join semilattice, but as will be seen later on, this is too strong for the general case.

Now we apply this to the analysis of example (2), which is repeated as (6) below.

(6) How many chairs does John have?

Three

P = ABS(how many chairs does john have?) $\leadsto$

$\lambda n|(\lambda y[*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)])| \geq n$

P is a finite set of natural numbers (we assume, in context, that there are only finitely many chairs, or finitely many **relevant** chairs). The order in *exh* is the natural order ≤ on numbers. <P, ≤ > is a join semilattice; it is easy to see that any subset Q of P is also a join semilattice.

T^1 = three $\leadsto \lambda P[P(3)] = \lambda P \exists n[n = 3 \wedge P(n)]$

T^2 = three $\leadsto \lambda n[n=3]$

$$exh(6) = |(\lambda y[*CHAIR(y) \wedge *HAVE(j,y)]| \geq 3 \wedge$$
$$\forall Q[[Q(3) \wedge Q \subseteq \lambda n|\lambda y(*CHAIR(y) \wedge *HAVE(j,y)]| \geq n] \rightarrow$$
$$maxQ \leq max(\lambda n[n=3] \cap \lambda n|\lambda y[*CHAIR(y) \wedge *HAVE(j,y)]| \geq n)]$$

In words: John has at least 3 chairs, and for every subset of numbers of chairs owned by John which contains 3, its largest member is smaller than or equal to the largest number in the intersection of {3} and the set of numbers of chairs owned by John.

Exh(6) means that John has exactly 3 chairs. The first main conjunct of exh(6) ensures that John has at least 3 chairs. I.e. the set of numbers of chairs owned by John is {1,2,3,…}. The second main conjunct of exh(6) requires that the largest number in every subset of this set is smaller than or equal to 3. Hence, John can not have more than 3 chairs. Let us consider 2 cases, a case where John has exactly 3 chairs, and a case where John has exactly 4 chairs.

Case 1: John has exactly 3 chairs

$[\![|\lambda y[*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)]|]\!] = 3$

$[\![\lambda n|\lambda y(*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0))| \geq n]\!] = \{1,2,3\}$

$[\![Q]\!] = \{1,2,3\}$; $[\![maxQ]\!] = 3$

$[\![max(\lambda n[n=3] \cap \lambda n|\lambda y[*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)]| \geq n])]\!] =$

$[\![max(\{3\} \cap \{1,2,3\})]\!] = 3$

Since $3 \leq 3$, exh(6) is true.

Case 2: John has exactly 4 chairs

$[\![|(\lambda y[*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)]|]\!] = 4$

$[\![\lambda n|\lambda y(*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0))| \geq n]\!] = \{1,2,3,4\}$

$[\![Q_1]\!] = \{1,2,3\}$; $[\![Q_2]\!] = \{1,2,3,4\}$

$[\![maxQ_1]\!] = 3$; $[\![maxQ_2]\!] = 4$

$[\![max(\lambda n[n=3] \cap \lambda n|\lambda y[*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)]| \geq n])]\!] =$

$[\![max(\{3\} \cap \{1,2,3,4\}]\!] = 3$

Since 4 is not smaller than or equal to 3, exh(6) is false.

(7) How many chairs does John have?

At least three

P = ABS(how many chairs does john have?) $\rightsquigarrow$

$\lambda n|(\lambda y[*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)])| \geq n$

T^1 = at least three $\rightsquigarrow \lambda P \exists n[n \geq 3 \wedge P(n)]$

T^2 = at least three $\rightsquigarrow \lambda n[n \geq 3]$

exh(7) = $|\lambda y[*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)]| \geq 3 \wedge$

$\forall Q[[\exists n[n \geq 3 \wedge Q(n)]) \wedge Q \subseteq \lambda n|\lambda y\ (*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)]| \geq n] \rightarrow$

$maxQ \leq max(\lambda n[n \geq 3] \cap \lambda n|\lambda y[*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)]| \geq n)]$

In words: John has at least 3 chairs, and for every subset of numbers of chairs owned by John which contains a number ≥ 3, its largest member is smaller than or equal to the largest number in the intersection between {3, 4, 5…} and the set of numbers of chairs owned by John.

Exh(7) means that John has at least 3 chairs. Let us check exh(7)'s truth condition in a state of affairs where John has exactly 4 chairs:

$[\![|\lambda y[*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0)]|]\!] = 4$

$[\![\lambda n|\lambda y(*CHAIR(y,w_0) \wedge *HAVE(j,y,w_0))| \geq n]\!] = \{1,2,3,4\}$

$[\![Q_1]\!] = \{1,2,3\}$; $[\![Q_2]\!] = \{1,2,3,4\}$

$[\![maxQ_1]\!] = 3$; $[\![maxQ_2]\!] = 4$

$[\![max(\lambda n[n\geq 3] \cap \lambda n|\lambda y[*CHAIR(y,w_0)\wedge *HAVE(j,y,w_0)]| \geq n)]\!] =$

$[\![max(\{3,4,5\ldots\}\cap\{1,2,3,4\}]\!] = [\![max(\{3,4\}]\!] = 4$

Since both 3 and 4 are smaller than or equal to 4, exh(7) is true.

Our reformulation of the exhaustivity operator helps us also in the following case:

(8) A: Who received you?

B: the assistant headmaster received me.

Bonomi and Casalegno (1993) observe that the reply in (9) below is ambiguous in the following way:

(9) A: Have you seen the headmaster?

B: No, only the assistant headmaster received me.

On one reading, exactly one person received me, and that person was the assistant headmaster. On the second reading, the assistant headmaster was the most important person who received me. It is the latter reading that interests us here.

Like the word *only*, the exhaustiveness operator has a different effect on non-ordered and ordered sets of alternatives. Also in example (8), we can distinguish between two cases. Suppose the set of the potential receivers consists of the headmaster, the assistant headmaster, secretary 1 and secretary 2. The members of this set can be naturally ordered on a scale of rank. I think that in interpreting B's answer we have a choice whether to use or ignore this ordering. If we ignore this ordering, we'll get the interpretation that the assistant headmaster received me, and no one else did. If we acknowledge the ordering, we'll get the interpretation that the assistant is the person with the highest rank who received me.

The first reading, where the rank ordering is ignored, works exactly the same way as the cases which were discussed in chapter 3. We use the plurality structure, and the formulation of *exh* in (5) reduces to the formulation in (1). Exh(8) in this case requires that the assistant headmaster received me, and that the members of every subset of plural receivers (closed under sum) which includes the assistant headmaster, are part of the individuals which are the assistant headmaster and come. Hence, the assistant headmaster received me, and no one else.

The second reading can be treated on a par with example (6). Let us restrict ourselves to C$\subseteq$ATOM, such that ranking is a linear order on C – for any two individuals, *a*, *b*, either *a* is ranked lower or equal to *b*, or higher than *b*. We can define the *max(a,b)* as the individual which is ranked higher, if there is such an individual. For two individuals with the same rank, max is undefined, hence a set of ranked individuals is not always closed under max. *Exh* requires that we look at all subsets of the set of receivers which are closed under max, and which include the headmaster. The assistant headmaster has to be the highest ranking individual in all these sets.

Now it is clear why we want the subsets of receivers Q to be closed under max. Let us look at some Q which does not include the headmaster, but includes the assistant headmaster, and an individual with the same rank, lets say, the deputy headmaster. That Q would not have a unique maximal element, and maxQ would be undefined. But (32), on its 'rank' reading, is intuitively true if both the assistant headmaster and the deputy headmaster received me. Closure under max leaves only these subsets Q which are **strictly** ordered by rank.

(10) Who received you?

[The assistant headmaster]$_F$ received me

P = ABS(who received you?) $\rightsquigarrow \lambda x[C(x) \wedge \text{RECEIVED}(x,i,w_0)]$; C is a background set of minimally two individuals ordered by rank. Thus, the order in *exh*, ≤, is the rank relation.

T^1 = the assistant headmaster $\rightsquigarrow \lambda P[P(\sigma(\lambda x AH(x,w_0))]$

T^2 = the assistant headmaster$\rightsquigarrow \lambda x[x=\sigma\ (\lambda x AH(x,w_0))]$

$$\mathrm{exh}(10) = [\mathrm{RECEIVED}(\sigma(\lambda x AH(x,w_0)),i,w_0) \wedge C(\sigma(\lambda x AH(x,w_0))] \wedge$$

$$\forall Q[[Q(\sigma(\lambda x AH(x,w_0)))\wedge Q \subseteq \lambda x[C(x) \wedge \mathrm{RECEIVED}(x,i,w_0)]] \rightarrow$$

$$\max Q \leq \max(\lambda x[x=\sigma(\lambda x AH(x,w_0))]\cap \lambda x[C(x) \wedge \mathrm{RECEIVED}(x,i,w_0)]]$$

Since the assistant headmaster received me, and since (s)he is a member of C, $\max(\lambda x[x=\sigma(\lambda x AH(x,w_0))]\cap\lambda x[C(x) \wedge \mathrm{RECEIVED}(x,i,w_0)]] = \sigma(\lambda x AH(x, w_0)$, and exh(10) can be reduced as follows:

$$\mathrm{exh}(10) = [\mathrm{RECEIVED}(\sigma(\lambda x AH(x,w_0)),i,w_0) \wedge C(\sigma(\lambda x AH(x,w_0))] \wedge$$

$$\forall Q[[Q(\sigma(\lambda x AH(x,w_0)))\wedge Q \subseteq \lambda x[C(x) \wedge \mathrm{RECEIVED}(x,i,w_0)]] \rightarrow$$

$$\max Q \leq \sigma(\lambda x AH(x, w_0)$$

In words: the assistant headmaster, who is a member of some set C, consisting of minimally two individuals ordered by rank, received me, and for every subset Q of individuals in C (s.t. members in Q are strictly ranked) who received me which includes the assistant headmaster, its highest ranking member is ranked lower than or equal to the assistant headmaster.

Exh(10) means that I was received by the assistant headmaster, who was the highest ranking individual in C who received me. Let us consider two cases: A case where the secretary, the assistant headmaster and the deputy headmaster received me, but not the headmaster, and a case where the assistant headmaster and the headmaster received me.

Case 1: The secretary, the assistant headmaster and the deputy headmaster received me, and no one else

$[\![C]\!]$ = {the secretary, the assistant headmaster, the deputy headmaster, the headmaster}

$[\![\leq]\!]$ = {the secretary ≤ the assistant headmaster, the deputy headmaster ≤ the headmaster}

$[\![\lambda x \mathrm{RECEIVED}(x,i,w_0))]\!]$ = {the secretary, the assistant headmaster, the deputy headmaster}

$[\![Q_1]\!]$ = {the assistant headmaster}; $[\![Q_2]\!]$ = {the secretary, the assistant headmaster}

$[\![\max(Q_1)]\!]$ = the assistant headmaster; $[\![\max Q_2]\!]$ = the assistant headmaster

Exh(10) is true in this model: the assistant headmaster ∈ {the secretary, the assistant h., the deputy h.}, and the assistant headmaster ≤ the assistant headmaster.

Case 2: The assistant headmaster and the headmaster received me, and no one else

$[\![C]\!]$ = {the secretary, the assistant headmaster, the deputy headmaster, the headmaster}

$[\![\leq]\!]$ = {the secretary ≤ the assistant headmaster, the deputy headmaster ≤ the headmaster}

$[\![\lambda x \mathrm{RECEIVED}(x,i,w_0))]\!]$ = {the assistant headmaster, the headmaster}

$[\![Q_1]\!]$ = {the assistant headmaster}; $[\![Q_2]\!]$ = {the assistant headmaster, the headmaster}

$[\![\max(Q_1)]\!]$ = the assistant headmaster; $[\![\max Q_2]\!]$ = the headmaster

Exh(10) is false in this model: the headmaster > the assistant headmaster

For a different order, let us look at a case with a transitive verb focus:

(11) What did you do with the letter?

I $[\text{typed}]_F$ it

(11) can be interpreted roughly as follows: The 'maximal' thing I did with the letter was type it (I might have formulated it, but I didn't mail it). The interpretation refers to some process of handling outgoing mail: first you formulate, then you type and then you mail. I'll analyze this example much in the same way I analyzed the 'rank' reading of (8), but in order for the exhaustivity operator to work in this case as well, I'll give a formulation for *exh* to allow it operate on properties and relations.

(12) Let C be a background set of properties (or relations) consisting minimally of 2 members, ordered by a partial order, $\leq$.

Let P, Q be variables ranging over **sets of properties** (or **relations**), partially ordered by $\leq$., s.t. $<Q, \leq>$ is a join semilattice.

We associate with VPs (or transitive verbs) two interpretations. An interpretation of the type of properties (or relations), and an interpretation of the type of sets of properties (or relations).

Let T be a variable over pairs of properties (or relations) and sets of properties (or relations).

If α is an expression of the type of pairs of properties (or relations) and sets of properties (or relations),

and $[\![\alpha]\!] = <\mathcal{T},\mathcal{P}>$, then $[\![\alpha^1]\!] = \mathcal{T}$ and $[\![\alpha^2]\!] = \mathcal{P}$.

$$exh = \lambda T\lambda P[APPLY(T^1,P) \wedge \forall Q[[APPLY(T^1,Q) \wedge Q \subseteq P] \rightarrow maxQ \leq max(T^2 \cap P)]$$

Explanatory note: In the domain of **plural individuals** we have $<T^1, T^2>$, where T^1 is a **generalized quantifier of plural individuals** and T^2 is a **set of plural individuals**. P and Q range over **sets of plural individuals**. In the domain of **properties** we have $<T^1, T^2>$, where T^1 is a **property** and T^2 is a **set of properties**. P and Q range over **sets of properties**. So I have reduced the general form such that I didn't introduce here a **generalized quantifier over properties**, but directly a property. This is for simplicity. The format can be generalized easily.

(13) What did you do with the letter?

I $[\text{typed}]_F$ it

P = ABS(what did you do with the letter?) $\rightsquigarrow \lambda P[P \subseteq C \wedge P(i, \sigma(\text{LETTER}))]$,

where P is a variable of the type of sets of relations.

C is an ordered background set of relations. In this case, C contains things one does with letters in the process of handling outgoing mail, such as *formulate*, *handwrite*, *type*, *print*, *mail*, *file a copy of* etc... C is partially ordered as follows: Let P, Q$\in$C, $P \preceq Q$ means something like "P is at least as early as Q in the process". For example, *formulate*$\preceq$ *type*$\preceq$ *mail*$\preceq$ *file a copy of* . The order $\leq$ in *exh*, will be this order, $\preceq$.

C is not closed under *max* (for example the max of *type* and *handwrite* is not defined, because these are not ranked relative to each other). As in the case of ranked individuals, subsets of C which are closed under max, are those which are strictly ordered.

T^1 = type $\rightsquigarrow$ TYPE

T^2 = type $\rightsquigarrow \lambda P[P=\text{TYPE}]$

$\text{APPLY}(T^1,P) = P(T^1) = \lambda P[P\subseteq C \wedge P(i, \sigma(\text{LETTER})](\text{TYPE}) =$

$\text{TYPE}(i, \sigma(\text{LETTER})) \wedge \text{TYPE} \subseteq C$

$T^2 \cap P = \lambda P[P=\text{TYPE}] \cap \lambda P[P\subseteq C \wedge P(i, \sigma(\text{LETTER})]$. Since TYPE is in the set

$\lambda P[P\subseteq C \wedge P(i, \sigma(\text{LETTER})]$, $T^2 \cap P = \{\text{TYPE}\}$

$\max(T^2 \cap P)] = \max(\{\text{TYPE}\}) = \text{TYPE}$

$\text{exh}(13) = \text{TYPE}(i, \sigma(\text{LETTER})) \wedge \text{TYPE} \subseteq C \wedge$

$$\forall Q[[Q(\text{TYPE}) \wedge Q \subseteq \lambda P[P\subseteq C \wedge P(i, \sigma(\text{LETTER})] \rightarrow \max Q \preceq \text{TYPE}]$$

In words: I typed the letter (and *type* is a member of a partially ordered set of relations, C, consisting of minimally two relations), and for every subset Q of relations in C between me and the letter, which includes *type* such that Q is strictly ordered - its maximal relation is at least as early in the process as *type*.

Exh(13) means that I typed the letter, and this was the most “advanced” thing out of the things in C that I did with it. Let us consider two cases: A case where I formulated and typed the letter, and didn’t mail it, and a case where I typed and mailed the letter.

Case 1: I formulated and typed but didn't mail the letter

$[\![C]\!]$ ={FORMULATE, TYPE, HANDWRITE, MAIL}

$[\![\leq]\!]$ = {FORMULATE $\preceq$ FORMULATE; FORMULATE $\preceq$ TYPE; FORMULATE $\preceq$ HANDWRITE; FORMULATE $\preceq$ MAIL, TYPE $\preceq$ TYPE, TYPE $\preceq$ MAIL, HANDWRITE $\preceq$ HANDWRITE; HANDWRITE $\preceq$ MAIL; MAIL $\preceq$ MAIL}

$[\![\lambda P[P(i, \sigma(\text{LETTER})]]\!]$ = {FORMULATE, TYPE}

$[\![Q_1]\!]$ = {TYPE}; $[\![Q_2]\!]$ = {FORMULATE,TYPE}

$[\![\max(Q_1)]\!]$ = TYPE; $[\![\max Q_2]\!]$ = TYPE

Exh(13) is true in this model. TYPE $\preceq$ TYPE

Case 2: I typed and mailed the letter

$[\![C]\!]$ ={FORMULATE, TYPE, HANDWRITE, MAIL}

$[\![\leq]\!]$ = {FORMULATE $\preceq$ FORMULATE; FORMULATE $\preceq$ TYPE; FORMULATE $\preceq$ HANDWRITE; FORMULATE $\preceq$ MAIL, TYPE $\preceq$ TYPE, TYPE $\preceq$ MAIL, HANDWRITE $\preceq$ HANDWRITE; HANDWRITE $\preceq$ MAIL; MAIL $\preceq$ MAIL}

$[\![\lambda P[P(i, \sigma(\text{LETTER})]]\!]$ = {TYPE, MAIL}

$[\![Q_1]\!]$ = {TYPE}; $[\![Q_2]\!]$ = {TYPE, MAIL}

$[\![\max(Q_1)]\!]$ = TYPE; $[\![\max Q_2]\!]$ = MAIL

Exh(13) is false in this model. MAIL $\npreceq$ TYPE

4.2 Exhaustivity on quasi-plural domains

Consider the following example with a VP focus:

(14) What did you do last night?

I [saw a movie]$_F$

Contrary to (13), there is no contextually salient ordering that we can use here in *exh*. Nevertheless (14) has an exhaustive interpretation, roughly described as follows: the only “interesting” thing that I did last night was see a movie (I didn’t go to a party, I didn’t write my dissertation etc…). Let us assume a background set of alternatives which includes the following properties: staying home, going out, going to the movies, going out for dinner, reading a book, and writing one’s dissertation. It seems that the sum of every two properties which do not contradict each other, or typically clash with one another in some way, is also considered an alternative (for example going to the movies **and** dining out). This is a kind of closure on the set of alternatives. I assume that if the background set of alternatives is not ordered by some contextual ranking, we always have the possibility to order it by a part-of relation . P $\sqsubseteq$ Q, in this case, means: if you have Q, then you have P.

Consider the following background set of properties: C = {*stay home, go out, stay home and write, stay home and read, stay home and see a movie, go out and see a movie, go out to a party, go out for dinner}*. C is not closed under sum formation. However, in some intuitive way, it seems to be a mixture of two background sets – We can build two different join semilattices out of it - the “algebra of staying home” whose atoms are: *write*, *read*, *see a movie*, and "the algebra of going out" whose atoms are: *see a movie*, *be at a party*, *eat dinner*. Each of these two sets of atoms can be closed under sum formation, and provide a suitable construction for *exh*.

Now back to the example.

(15) What did you do (last night)?

I [saw a movie]$_F$

P = ABS(what did you do?) $\rightsquigarrow \lambda P[(P \subseteq C \wedge P(i)]$, where P is a variable of the type of sets of properties, and C is a background set of properties, closed under sum formation.

$\leq$ in *exh* is the part of relation between properties, $\sqsubseteq$.

T^1 = saw a movie $\rightsquigarrow$ SM

T^2 = saw a movie $\rightsquigarrow \lambda P[P= SM]$

APPLY(T^1,P) = $P(T^1) = \lambda P[P \subseteq C_1 \wedge P(i)](SM) = SM(i) \wedge (SM \subseteq C)$

$T^2 \cap P = \lambda P[P= SM] \cap \lambda P[P \subseteq C \wedge P(i)]$. Since SM is in the set $\lambda P[P \subseteq C \wedge P(i)]$,

$T^2 \cap P = \{SM\}$

$\max(T^2 \cap P)] = \max(\{SM\}) = SM$

exh(15) = $SM(i) \wedge (SM \subseteq C) \wedge \forall Q[[Q(SM) \wedge Q \subseteq \lambda P[P \subseteq C \wedge P(i)] \rightarrow \max Q \sqsubseteq SM]$

In words: I saw a movie (and *seeing a movie* is a member of a set of properties C, consisting of minimally two elements and closed under sum formation), and for every subset of properties in C (closed under sum formation), which I have, and which includes *seeing a movie*, its maximal property is a part of *seeing a movie*.

Exh(15) means that out of the properties in C, I only have *seeing a movie* and the properties which are part of it.

Let us assume first that we're in the 'algebra of going out', the atoms of C are: *see a movie*, *eat dinner*. Let us consider 2 cases, a case were (I went out and) saw a movie, but didn't have dinner, and a case where (I went out and) saw a movie, and ate dinner.

<u>Case 1: I went out: I saw a movie and didn't have dinner</u>

$[\![C]\!]$ ={see a movie, have dinner, see a movie&have dinner}

$[\![\lambda P[P(i)]\ ]\!]$ = {see a movie}

$[\![Q]\!]$ = {see a movie}

$[\![max(Q)]\!]$ = see a movie

Exh(15) is true in this model. see a movie $\sqsubseteq$ see a movie

<u>Case 2: I went out: I saw a movie and had dinner</u>

$[\![C]\!]$ ={see a movie, have dinner, see a movie&have dinner}

$[\![\lambda P[P(i)]\ ]\!]$ = ={see a movie, have dinner}

$[\![Q_1]\!]$ = {see a movie}, $[\![Q_2]\!]$ = {see a movie, have dinner, see a movie&have dinner}

$[\![\max(Q_1)]\!]$ = see a movie; $[\![\max(Q_2)]\!]$ = see a movie&have dinner

Exh(15) is false in this model. see a movie&have dinner $\not\sqsubseteq$ saw a movie

So, in the 'algebra of going out' the conversation implicates that I didn't dine out. In the 'algebra of staying in', assuming the atoms of C are: *see a movie*, *read a book*, the implicature would be that I didn't read a book.

The case of a determiner focus can be analyzed in a similar way.

(16) A: How many men come?

B: $[\text{Most}]_F$ men come.

The abstract of the question, *how many men come?*, was analyzed as a set of cardinalities – the set of numbers such that at least that many men come (see examples 6 and 7 in section 4.1). However, the short answer in this case, *most*, is not a set of sets of cardinalities (as we assumed for numerical expressions), but a relation between cardinalities. *Most men come* means that the number of men who come is larger than the number of men who don't come. So, B's reply really answers the following question: *what is the set of relations that hold between the number of men who come and the number of men who don't come?* For simplicity, I'll assume that A's question can be interpreted in that way.

I assume that natural language determiners are relations between cardinalities:

(17) Let P, Q $\subseteq$ D, let n, m be numbers s.t. m = |P$\cap$Q| and n = |P-Q|

all(m,n) = $\lambda m \lambda n[n=0]$

most(m,n) = $\lambda m \lambda n[m > n]$

many(m,n) = $\lambda m \lambda n[m > p \cdot n]$, where $0<p\leq 1$

some(m,n) = $\lambda m \lambda n[m \neq 0]$

no(m,n) = $\lambda m \lambda n[m=0]$

few(m,n) = $\lambda m \lambda n[m < p \cdot n]$, where $0<p\leq 1$

not all(m,n) = $\lambda m \lambda n[n \neq 0]$

We begin by defining a **part-of** relation between determiners:

(18) Let R_1, R_2 be relations between cardinalities, and let Let P, Q $\subseteq$ D, $R_1 \sqsubseteq R_2$ (R_1 is a part of R_2) iff for every cardinality m = |P$\cap$Q| s.t P$\neq\emptyset$ and n = |P-Q|, if R_2(m,n), then R_1(m,n).

(19) Let $\mathcal{R}$ be a set of relations, max($\mathcal{R}$) is the relation in $\mathcal{R}$ (if it exists), such that for every R in $\mathcal{R}$: R $\sqsubseteq$ max($\mathcal{R}$)

For example, s*ome* is a part of *most* (*Most P's are Q's* entails *some P's are Q's*), and of *many*. *Many* is a part of *most* (*many* has an interpretation such that *most P's are Q's* entails *many P's are Q's*). *Few* is a part of *no* (*no P's are Q's* entails that *few P's are Q's)*. Note that *some, many*, and *most* **are** parts of *all* (all P's are Q's entails that some, many or most P's are Q's, if there are P's), and similarly, *not all* **is** a part of *no*.

As with properties, sets of determines aren't generally closed under sum formation – there is no sum of *some* and *no*, there is no sum of *all* and *not all*. Hence a background set of determiners doesn't always provide us with a suitable construction for *exh*. However, there is a general way to split the set of determiners into two subsets such that these subsets would be closed under sum. So, where in the case of properties we had an "algebra of staying in", and an "algebra of going out", here we have an algebra of "positive" determiners, and an algebra of "negative" ones.

Each of the sets {some, many, most, all} and {not every, few, no} is closed under sum formation, and is "perfect" as background sets in the definition of *exh*. As a matter of fact, these are exactly the Horn scales for determiners.

Remember that one of the problems I brought up for Horn's theory is the stipulative nature of the scale, and in particular of the restriction to elements that are all "positive" or all "negative". In an approach like Horn's that restriction ought to derive from the pragmatic theory, but it is not clear at all that it does. In the present theory, the separation into "positive" and "negative" scales is derived from the semantics of exhaustivization: *exh* requires the domain it operates on to be ordered as a join semilattice. Domains with both "positive" and "negative" determiners cannot satisfy this requirement, while homogenously positive or homogenously negative domains can. Here, exhaustivization **itself** brings about the natural partition of the domain of determiners into the Horn scales, and hence **derives** the scales.

Thus, the present theory actually explains why we find the particular Horn scales we find, when we find them.

Lets us consider a few examples,

(20) How many men come?

$[\text{Some}]_F$ men come.

P = ABS(how many men come?) $\rightsquigarrow \lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN\text{-}COME|)]$,

where P is a variable of the type of sets of relations between cardinalities, and C is a background set of relations between cardinalities, which contains at least two members, and which is closed under sum.

$\leq$ in *exh* is the part-of relation between relations between cardinalities, $\sqsubseteq$. P stands for the set of relations that hold between the number of men who come and the number of men who don't come.

T^1 = some $\rightsquigarrow \lambda m\lambda n[m \neq 0]$

T^2 = some $\rightsquigarrow \lambda P[P = \lambda m\lambda n[m \neq 0]]$

$APPLY(T^1,P) = P(T^1) = (\lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN\text{-}COME|)])(\lambda m\lambda n[m \neq 0])$

$= |MAN \cap COME| \neq 0 \wedge \lambda m\lambda n[m \neq 0] \subseteq C$

$(T^2 \cap P) = \lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN\text{-}COME|)] \cap \lambda P[P = \lambda m\lambda n[m \neq 0]]$,

since $\lambda m\lambda n[m \neq 0]$ is in the set $\lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN\text{-}COME|)]$, $T^2 \cap P = \{\lambda m\lambda n[m \neq 0]\}$ and $\max(T^2 \cap P) = \lambda m\lambda n[m \neq 0]$

$$\text{exh}(20) = |\text{MAN}\cap\text{COME}| \neq 0 \wedge \lambda m\lambda n[m\neq 0] \subseteq C \wedge$$

$$\forall Q[Q(\lambda m\lambda n[m\neq 0]) \wedge Q \subseteq \lambda P[P\subseteq C \wedge P(|\text{MAN}\cap\text{COME}|, |\text{MAN-COME}|)]] \rightarrow$$

$$\text{max}Q \sqsubseteq \lambda m\lambda n[m\neq 0]]$$

In words: Some men come and *some* is in a set of relations, C, consisting minimally of two elements, and for every subset of relations in C (closed under sum formation) between the cardinalities of men who come and men who don't come, which includes *some*, its maximal element is part of *some*.

If the background set of relations C is {some, most, all), then exh(20) means that some, but not most or all men come. Let us check its truth conditions in three cases, a case where 4 out of 4 men come, a case where 3 out 4 men come, and a case where 2 out of 4 men come.

<u>Case 1: all men come</u>

$[\![C]\!] = \{\text{SOME} \sqsubseteq \text{MOST} \sqsubseteq \text{ALL}\}$

$|\text{MAN}\cap\text{COME}| = 4$; $|\text{MAN - COME}| = 0$

$[\![\lambda P[P\subseteq C\wedge P(|\text{MAN}\cap\text{COME}|, |\text{MAN}\cap\text{COME}|)]]\!] = \{\text{SOME, MOST, ALL}\}$

$[\![Q_1]\!] = \{\text{SOME}\}$; $[\![Q_2]\!] = \{\text{SOME, MOST}\}$, $[\![Q_3]\!] = \{\text{SOME, MOST, ALL}\}$

$[\![\max(Q_1)]\!] = \text{SOME}$; $[\![\max Q_2]\!] = \text{MOST}$, $[\![\max Q_3]\!] = \text{ALL}$

Exh(20) is false in this model. MOST,ALL$\not\sqsubseteq$ SOME.

Case 2: most but not all men come

$[\![C]\!] = \{SOME \sqsubseteq MOST \sqsubseteq ALL\}$

$|MAN \cap COME| = 3; |MAN - COME| = 1$

$[\![\lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN \cap COME|)]]\!] = \{SOME, MOST\}$

$[\![Q_1]\!] = \{SOME\}; [\![Q_2]\!] = \{SOME, MOST\},$

$[\![max(Q_1)]\!] = SOME; [\![maxQ_2]\!] = MOST$

Exh(20) is false in this model. $MOST \not\sqsubseteq SOME$.

Case 3: some but not most men come

$[\![C]\!] = \{SOME \sqsubseteq MOST \sqsubseteq ALL\}$

$|MAN \cap COME| = 2; |MAN - COME| = 2$

$[\![\lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN \cap COME|)]]\!] = \{SOME\}$

$[\![Q]\!] = \{SOME\}\}$

$[\![max(Q)]\!] = SOME$

Exh(20) is true in this model. $SOME \sqsubseteq SOME$.

It is important to note that the background set C is crucial for calculating the implicatures in this case. If C={some, all} the only implicature is that not all men come, if C={some, most, all} the implicatures are that not all men come, and that not most men come. I believe that the dependency on the background set, C, captures a real fact concerning the implicatures of *[Some]$_F$ men come*. Some of the implicatures are especially ‘fuzzy’. It is not clear whether the sentence really implicates that it is not the case that most men come, or that it is not the case that many men come, etc… If the context hints that *many* is in C, such as in (21) below, the implicature *not many* is more robust:

(21) –I heard that many students failed the exam

-[some]$_F$ failed.

The implicature *not all* is quite robust. Even if *all* itself is not in C, the implicatures *not most* or *not many* entail the implicature *not all*.

A nice result of my analysis is that the implicatures of *[some]$_F$ men came* are not the same as the implicatures of *[one or more]$_F$ men came*. The exhaustivization of *[one or more]$_F$ men came* means simply that at least one man came (the computation is the same as in example 7). The crucial difference between the cases is the interpretations of the question (How many men came?), and the nature of the ordering we used in *exh*. In the case of *one or more* the question abstract is interpreted as a set of numbers, and thus *one or more* is interpreted as a generalized quantifier over numbers - $\lambda P \exists n[n \geq 1 \wedge P(n)]$. The ordering was the natural ordering of numbers. In the case of s*ome*, we interpreted the question abstract as a set of relations. The natural order on numbers does easily provide a suitable construction for *exh*. In the case of determiners, we had to split the domain into two in order to get the right construction.

A question that presents itself now is whether we have the option to interpret numerals as determiners. Sets of numbers are just an instance of relations between numbers, and as a matter of fact, out of the set of determiners *most*, *many* and *few* are the only real relations between numerals. Let us consider (22) below:

(22) A: [At least ten]$_F$ men came

B: [Some]$_F$ men came

What does B implicate? I think that (22) could be understood in two ways. Either B implicates that she does not know how many men came or that she knows that less than 10 men came. B's claim, without exhaustivization, is that at least one man came. Thus, if we do not interpret B's utterance exhaustively, it does not increase nor contradict the information conveyed in A's utterance. The only reason for B to do so, I think, is that B doesn't agree with A, but also doesn't know how many men came. A second way to understand B's utterance is to assume that B is including the determiner interpretation of *at least ten* in the background set C of relations. B is treating A's utterance as a possible answer to *what is the relation that holds between the number of men who come and the number of men who don't come?*, and contrasts her answer to that question, *some*, with A's answer *at least 10*. The exhaustivization of B's utterance relative to a set that contains the relation *at least 10*, entails that it is not the case that at least 10 men came. Thus, if we assume this is the case, B's utterance implicates that less than 10 men came. I think this is a special case, where the set C contains only the two determiners *some* and *at least 10*. In this case, we even don't implicate *not all*.

I end this section by showing that relative to a set of alternatives {not every, no}, the sentence *[Not all]$_F$ men come* as an answer to *How many men come?* implicates that some men come.

(23) How many men come?

[Not all]$_F$ men come.

P = ABS(how many men come?) $\rightsquigarrow \lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN\text{-}COME|)]$, where P is a variable of the type of sets of relations between cardinalities, and C is a background set of relations between cardinalities, which contains at least two members, and which is closed under some.

$\leq$ in *exh* is the part-of relation between relations between cardinalities, $\sqsubseteq$. P stands for the set of relations that hold between the number of men who come and the number of men who don't come - I assume that P is closed under sum formation.

T^1 = not all $\rightsquigarrow \lambda m \lambda n[n \neq 0]$

T^2 = not all $\rightsquigarrow \lambda P[P = \lambda m \lambda n[n \neq 0]]$

APPLY(T^1,P) = P(T^1) = $(\lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN\text{-}COME|)])(\lambda m \lambda n[n \neq 0])$

$= |MAN\text{-}COME| \neq 0 \wedge \lambda m \lambda n[n \neq 0] \subseteq C$

$(T^2 \cap P) = \lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN\text{-}COME|)] \cap \lambda P[P = \lambda m \lambda n[n \neq 0]]$,

since $\lambda m \lambda n[n \neq 0]$ is in the set $\lambda P[P \subseteq C \wedge P(|MAN \cap COME|, |MAN\text{-}COME|)]$, $T^2 \cap P$ = $\{\lambda m \lambda n[n \neq 0]\}$ and $\max(T^2 \cap P) = \lambda m \lambda n[n \neq 0]$

$$\text{exh}(23) = |\text{MAN-COME}| \neq 0 \wedge \lambda m\lambda n[n\neq 0] \subseteq C \wedge$$
$$\forall Q[Q(\lambda m\lambda n[n\neq 0])\wedge Q \subseteq \lambda P[P\subseteq C\wedge P(|\text{MAN}\cap\text{COME}|, |\text{MAN-COME}|)]] \rightarrow$$
$$\max Q \sqsubseteq \lambda m\lambda n[n\neq 0]]$$

In words: Some men don't come, and *not all* is in a set of relations, C, consisting minimally of two elements, and for every subset of relations in C (closed under sum formation) between the cardinalities of men who come and men who don't come, which includes *not all*, its maximal element is part of *not all*.

If the background set of relations C is {not all, no), then exh(23) means that some, but not all men come. Let us check its truth conditions in two cases, a case where 0 out of 4 men come, a case where 2 out 4 men come.

<u>Case 1: no men come</u>

$[\![C]\!] = \{\text{NOT ALL} \sqsubseteq \text{NO}\}$

$|\text{MAN}\cap\text{COME}| = 0$; $|\text{MAN - COME}| = 4$

$[\![\lambda P[P\subseteq C\wedge P(|\text{MAN}\cap\text{COME}|, |\text{MAN}\cap\text{COME}|)]]\!] = \{\text{NOT ALL, NO}\}$

$[\![Q_1]\!] = \{\text{NOT ALL}\}$; $[\![Q_2]\!] = \{\text{NOT ALL, NO}\}$

$[\![\max(Q_1)]\!] = \text{NOT ALL}$; $[\![\max Q_2]\!] = \text{NO}$

Exh(23) is false in this model. $\text{NO} \not\sqsubseteq \text{NOT ALL}$.

Case 2: some but not all men come

$[\![C]\!] = \{\text{NOT ALL} \sqsubseteq \text{NO}\}$

$|\text{MAN} \cap \text{COME}| = 2$; $|\text{MAN} - \text{COME}| = 2$

$[\![\lambda P[P \subseteq C \wedge P(|\text{MAN} \cap \text{COME}|, |\text{MAN} \cap \text{COME}|)]]\!] = \{\text{NOT ALL}\}$

$[\![Q]\!] = \{\text{NOT ALL}\}$

$[\![\max(Q)]\!] = \text{NOT ALL}$

Exh(23) is true in this model. NOT ALL $\sqsubseteq$ NOT ALL.

Summing up so far, the exhaustivity operator makes use of join semilattices. I.e. ordered sets, closed under a maximality operator. The domain of plural and individual entities, provides such constructions. So do finite sets of numbers, and any strictly totally ranked set. *Exh* can operate on other domains as well, with the assumption that a suitable construct can be built in context. The case of determiners is special in that there are few of them, and that there is an easy way to group them into two suitable sets.

The careful reader must have noticed that I haven't dealt yet with yes/no questions. I'll discuss them in chapter 6. In the next chapter I show how the exhaustivity analysis of scalar implicatures explains in a straightforward way their 'projection' behavior.

Chapter 5

The Projection of Scalar Implicatures

In this chapter I discuss the phenomenon of scalar implicature ‘projection’. Section 5.1 deals with ‘embedded’ scalar implicatures, i.e. scalar implicatures that show up in the scope of embedding operators, and section 5.2, with the suspension of scalar implicatures in the scope of downward entailing (DE) operators. I briefly present previous analyses of these phenomena (Landman (2000) and Chierchia (ms)), and discuss how these cases are dealt within the exhaustivity theory of scalar implicatures, which was introduced in the previous chapter. Section 5.3 deals with other environments in which scalar implicatures are ‘suspended’ or ‘cancelled’.

5.1 Embedded scalar implicatures

One of the main problems of Horn’s analyses of scalar implicatures is that it doesn’t predict the correct facts about implicatures that arise under the scope of embedding operators (see also discussion in chapter 1, section 1.7). Landman (2000) and Chierchia (ms) discuss, among others, examples such as (1)-(3):

(1) Bill knows that there were 3 boys at the party

(2) Every boy kissed 3 girls

(3) Some boy kissed 3 girls

In many contexts, sentences (1)-(3) are understood as conveying (4)-(6) respectively. These interpretations cannot be inferred on the basis of the implicatures that Horn predicts for (1)-(3), given in (7)-(9).

(4) Bill knows that there were exactly 3 boys at the party

(5) Every boy kissed exactly 3 girls

(6) Some boy kissed exactly 3 girls

(7) It is not the case that Bill knows that there were 4 boys at the party

(8) It is not the case that every boy kissed 4 girls

(9) It is not the case that some boy kissed 4 girls = No boy kissed 4 girls

Sentence (1) can be understood as conveying that Bill knows that there were exactly 3 boys at the party. The implicature that Horn predicts for this sentence is much too weak, it is compatible with Bill not knowing how many boys were at the party. Sentence (2) has an interpretation that every boy kissed exactly 3 girls. Again, the implicature derived by Horn is too weak. (8) is compatible with some boys kissing more than 3 girls. On the other hand, the implicature that Horn predicts for (3) is too strong. (3) is typically understood as conveying (6), (9) is much too strong.

One of the problems is that on Horn's analysis of scalar implicatures, negation has to take scope over the whole sentence, and thus, in the examples discussed above, it ends up in the wrong place. If we allow implicatures to be computed locally, we get the right results for (1) and (2). Sentences (1)-(2) would implicate (10)-(11) respectively.

(10)-(11) are the right implicatures, in the sense that together with the meanings they entail the desired interpretations. Note that this isn't enough for (3). The right implicature of (3) is (12') , and not (12). Here we have an additional problem – the anaphoric reference of the implicature to the meaning.

(10) Bill knows that it is not the case that there were 4 boys at the party

(11) For every boy, it is not the case that he kissed 4 girls

(12) For some boy, it is not the case that he kissed 4 girls

(12') For *that* boy (i.e. the one mentioned in example 3) , it is not the case that he kissed 4 girls.

Both Landman (2000) and Chierchia (ms) suggest that implicatures such as the above are computed by the grammar. Landman deals specifically only with the case of the 'exactly' implicature of numbers, while Chierchia restricts himself to 'default' or 'generalized conversational implicatures', implicatures which usually arise, unless cancelled explicitly (such as the exclusive interpretation of *or* and the *not all* interpretation of *some*).

Landman's general idea is that a scalar implicature is introduced locally, and inherits up following the semantic composition of the sentence. For example in the case of (2), which is repeated in (13) below, the implicature is introduced at the level of the VP interpretation, *kissed 3 girls*. The implicature of the VP is (14):

(13) Every boy kissed 3 girls

(14) {x: x kissed no more than 3 girls}

In the process of inheriting up, we apply (14) to the interpretation of *every boy*, and we get (15):

(15) Every boy kissed no more than 3 girls

Chierchia (ms) has a similar theory in which grammar computes two semantic values for each expression: a ‘plain’ semantic value (the meaning) and a ‘strengthened’ semantic value (the meaning strengthened by the scalar implicature). Chierchia, too, assumes that implicatures are introduced locally in ‘the scope domain’ of the scalar element. I’ll show how Chierchia’s theory works using example (16) below.

(16) Some boy kissed 3 girls

We work bottom up in the syntactic tree. The first scalar element which we encounter is *3*. The scope domain of 3 is the VP *kissed 3 girls*. The first step in calculating strengthened semantic values is specifying the set of relevant alternatives for our expression. This is done by substituting the scalar element in the expression with all elements in the Horn scales which our scalar element is part of. The set of alternatives for *kissed 3 girls* is{*kissed 1 girl*, *kissed 2 girls, kissed 3 girls*, *kissed 4 girls, kissed 5 girls*…}. Out of this set, we pick up the one which is immediately stronger than our expression. In this case it is *kissed 4 girls*. The strengthened semantic value of *kissed 3 girls* is *kissed 3 girls and not kissed 4 girls*.

Now we move up the tree to the scalar element *some*. Its scope domain is *some boy kissed 3 girls*. Chierchia assumes that when specifying the alternatives for an expression that contains two scalar elements, one embedded in the scope of the other, we ignore the alternatives for the embedded element (these were already taken care of before). The alternatives for *some boy kissed 3 girls* are {*some boy kissed 3 girls*, *most boys kissed 3 girls*, *every boy kissed 3 girls*}. The one which is immediately stronger than our target is *most boys kissed 3 girls*. The contribution of *some* to the strengthened semantic value of the sentence would be *some boy kissed 3 girls, and not most boys kissed 3 girls.* We get the strengthened semantic value of the whole sentence by adding this to the strengthened semantic value of *kissed 3 girls* applied to the subject NP, *some boy*. Thus, the strengthened semantic value of *some boy kissed 3 girls* is *Some boy kissed exactly 3 girls and it is not the case that most boys kissed 3 girls*.

I'll show now that the facts about 'embedded' implicatures are naturally explained if scalar implicatures are analyzed as exhaustivity effects. After that, in sections 5.1.2 and 5.2, I will come back to the proposals of Landman and Chierchia.

5.1.1 Embedding under *every*

Let us consider the following two examples:

(17) Whom did every boy kiss?

Every boy kissed [3 girls]$_F$

(18) How many girls did every boy kiss?

Every boy kissed $[3]_F$ girls.

Both (17) and (18) are ambiguous. Concerning (17), on the reading in which *whom* takes wide scope relative to *every boy*, the most natural interpretation of the answer is that there are exactly 3 girls who were kissed by every boy, and that no one else was kissed by every boy. On the reading in which *every boy* takes wide scope relative to *whom*, the most natural interpretation of the answer is that every boy kissed exactly 3 girls and no one else. Similarly, (18) is ambiguous between a reading in which there were exactly 3 girls who were kissed by every boy (and there's no implicature concerning entities other than girls who were or were not kissed by every boy), and a reading in which every boy kissed exactly 3 girls (not necessarily the same girls).

Let me start with (17). I'll do first the case where *whom* takes scope over *every boy*, and accordingly, in the answer, *3 girls* takes scope over *every boy*. The relevant interpretation here is that there are exactly 3 girls which were kissed by every boy, and there is no one else who was kissed by every boy. This case will be analyzed much in the same way as example (17), chapter 3. I omit the references to w_0 in order to make the formulas more readable. For the reader's convenience, I repeat here the relevant formulation of exhaustivity.

(19) Let P and Q be variables ranging over sets of the form *X for some $X \subseteq ATOM$.

We associate with noun phrases two interpretations. NP_{ARG} of type <<e,t>,t> and NP_{PRED} of type <e,t>. Let T be a variable of type <<e,t>,t>×<e,t> (a variable over pairs of sets of sets and sets). If $\alpha \in EXP_{<<e,t>,t>\times<e,t>}$ and $[\![\alpha]\!] = <\mathcal{T},\mathcal{P}>$, then $[\![\alpha^1]\!] = \mathcal{T}$ and $[\![\alpha^2]\!] = \mathcal{P}$.

$$exh = \lambda T\lambda P[T^1(P)\wedge\forall Q[[T^1(Q)\wedge Q \subseteq P] \rightarrow \sigma Q \sqsubseteq \sigma(T^2\cap P)]$$

(17) Whom did every boy kiss?

Every boy kissed [3 girls]$_F$

P= ABS(whom did every boy kiss?) $\rightsquigarrow \lambda y[BOY \subseteq \lambda x{*}KISS(x,y)]$

T^1= three girls $\rightsquigarrow \lambda P[\exists y\in(\lambda y{*}GIRL(y): |y| = 3 \wedge P(y))]$

T^2=BE(three girls) = $\lambda y[{*}GIRL(y) \wedge |y|=3]$

$$exh(17) = \exists y\in[\lambda y{*}GIRL(y): |y| = 3 \wedge BOY \subseteq \lambda x{*}KISS(x,y)] \wedge$$
$$\forall Q[[\exists y\in(\lambda y{*}GIRL(y): |y| = 3 \wedge Q(y))] \wedge Q \subseteq \lambda y[BOY \subseteq \lambda x{*}KISS(x,y)]] \rightarrow$$
$$\sigma Q \sqsubseteq \sigma(\lambda y[{*}GIRL(y) \wedge |y|=3]\cap \lambda y[BOY \subseteq \lambda x{*}KISS(x,y)])]$$

In words: there is a sum of girls with exactly 3 atomic elements whom every boy kissed, and for every subset of plural individuals who were kissed by every boy, and which includes a sum of girls with exactly 3 atomic elements, its sum is a part of the girls with exactly 3 atomic elements who were kissed by every boy.

Exh(17) means that there are exactly 3 girls who were kissed by every boy, and there is no one else who was kissed by every boy. The first main conjunct of exh(17) ensures that there are at least 3 girls whom every boy kissed. The second main conjunct ensures that there are only 3 girls that every boy kissed, and no one else was kissed by every boy, here's why. In order for $\sigma(\lambda y[{}^*GIRL(y) \wedge |y|=3] \cap \lambda y[BOY \subseteq \lambda x{}^*KISS(x,y)])$ to be defined, the set $(\lambda y[{}^*GIRL(y) \wedge |y|=3] \cap \lambda y[BOY \subseteq \lambda x{}^*KISS(x,y)])$ has to be a singleton set. That means that there has to be only one plural sum of girls with 3 atomic elements that was kissed by every boy, i.e., there cannot be 4 girls that were kissed by every boy. Further more, the formula is false if there are exactly 3 girls and some one which is not a girl who were kissed by every boy. Let us assume that Mary, Sarah, Sue and Bill were kissed by every boy. Now, of the subsets, Q is the set Mary, Sarah, Sue, Bill, Mary⊔Sarah, Mary⊔Sue, Mary⊔Bill, Sarah⊔Sue, Harry⊔Bill, Bill⊔Sue, Mary⊔Sarah⊔Sue, Mary⊔Sue⊔Bill, Sarah⊔Sue⊔Bill, Mary⊔Bill⊔Sarah, Mary⊔Sarah⊔Sue⊔Bill}. The sum of this set, Mary⊔Sarah⊔Sue⊔Bill, is not part of the girls with exactly 3 atomic elements who were kissed by every boy (Mary⊔Sarah⊔Sue).

Now we look at the case where *every boy* takes scope over *whom*, so the interrogative sentence is interpreted as *for every boy, whom did he kiss?*. Accordingly, in the answer, *every boy* takes scope over *3 girls*. The relevant interpretation here is that for every boy, there are exactly 3 girls that he kissed, and that no boy kissed something which isn't a girl.

The interrogative sentence in this case is not a simple one place constituent question, but rather a constituent question in the scope of a universal quantifier. I assume that the exhaustivity operator too, that operates on the question abstract will take scope under the universal quantifier. The question abstract, $\lambda y{*}KISS(x,y)$, contains a free variable x, which will be bound by the universal quantifier from the 'outside'.

(17') Whom did every boy kiss?

Every boy kissed [3 girls]$_F$

P= ABS(whom did x kiss?) $\rightsquigarrow \lambda y{*}KISS(x,y)$

T^1= three girls $\rightsquigarrow \lambda P[\exists y \in \lambda y{*}GIRL(y): |y| = 3 \wedge P(y)]$

T^2=BE(three girls) = $\lambda y[{*}GIRL(y) \wedge |y|=3]$

exh(17') = For every boy x, $\{\exists y \in [\lambda y{*}GIRL(y): |y| = 3 \wedge {*}KISS(x,y)] \wedge$

$$\forall Q[[\exists y \in \lambda y{*}GIRL(y): |y| = 3 \wedge Q(y)] \wedge Q \subseteq \lambda y{*}KISS(x,y)]] \rightarrow$$

$$\sigma Q \sqsubseteq \sigma(\lambda y[{*}GIRL(y) \wedge |y|=3] \cap \lambda y{*}KISS(x,y)])]\}$$

In words: For every boy, there is a sum of girls with exactly 3 atomic elements whom he kissed, and for every subset of plural individuals who were kissed by him, and which includes a sum of girls with exactly 3 atomic elements, its sum is a part of the girls with exactly 3 atomic elements whom he kissed.

Exh(17') means that every boy kissed exactly 3 girls and no one else. I'll leave it to the reader to check this by herself.

Now we move to (18). The reading in which *how many girls* takes wide scope relative to *every boy* will be analyzed much in the same way as example (6), chapter 4. The question is interpreted as *Which number(s) is/are such that every boy kissed at least that many* *girls?* The exactly effect will come through exhaustivization. For the reader's convenience, I repeat here the relevant formulation of exhaustivity.

(20) Let P, Q be variables ranging over sets, partially ordered by $\leq$., s.t. $<Q, \leq>$ is a join semilattice.

We associate with NPs or numerals, two interpretations: an interpretation of type $<<e,t>,t>$ and an interpretation of type $<e,t>$.

Let T be a variable of type $<<e,t>,t>\times<e,t>$ (a variable over pairs of sets of sets and sets). If $\alpha \in EXP_{<<e,t>,t>\times<e,t>}$ and $[\![\alpha]\!] = <\mathcal{T},\mathcal{P}>$, then $[\![\alpha^1]\!] = \mathcal{T}$ and $[\![\alpha^2]\!] = \mathcal{P}$.

$$exh = \lambda T\lambda P[T^1(P)\wedge\forall Q[[T^1(Q)\wedge Q \subseteq P] \rightarrow maxQ \leq max(T^2\cap P)]$$

(18) How many girls did every boy kiss?

Every boy kissed $[3]_F$ girls.

P = ABS(how many girls did every boy kiss?) $\rightsquigarrow$

$\lambda n|(\lambda y[*GIRL(y) \wedge [BOY \subseteq \lambda x*KISS(x,y)]])| \geq n$

T^1 = three $\rightsquigarrow \lambda P[P(3)] = \lambda P\exists n[n = 3 \wedge P(n)]$

T^2 = three $\rightsquigarrow \lambda n[n=3]$

$$\begin{aligned} exh(18) = {} & |(\lambda y[*GIRL(y) \wedge [BOY \subseteq \lambda x*KISS(x,y)]]| \geq 3 \wedge \\ & \forall Q[[Q(3)\wedge Q \subseteq \lambda n|\lambda y(*GIRL(y) \wedge[BOY\subseteq \lambda x*KISS(x,y)]]| \geq n] \rightarrow \\ & maxQ \leq max(\lambda n[n=3] \cap\lambda n|\lambda y[*GIRL(y) \wedge[BOY\subseteq \lambda x*KISS(x,y)]]| \geq n)] \end{aligned}$$

In words: The number of girls which every boy kissed is equal or larger than 3, and for every subset of numbers of girls kissed by every boy which contains 3, its largest member is smaller than or equal to the largest number in the intersection of {3} and the set of numbers of girls kissed by every boy.

Exh(18) means that there were exactly 3 girls kissed by every boy. The first main conjunct of exh(18) ensures that there were at least 3 girls who were kissed by every boy. I.e. the set of numbers of girls kissed by every boy is {1,2,3,…}. The second main conjunct of exh(18) requires that the largest number in every subset of this set is smaller than or equal to 3. Hence, there cannot be more than 3 girls kissed by every boy.

On the reading in which *every boy* takes scope over *how many girls*, I assume that the interpretation of the interrogative sentence is *for every boy, how many girls did he kiss?*. As in (17'), the interrogative sentence in this case too, is not a question, but a question in the scope of a universal quantifier. Again, I assume that the exhaustivity operator, which operates on the question abstract, takes scope under the universal quantifier.

(18') How many girls did every boy kiss?

Every boy kissed $[3]_F$ girls.

P = ABS(how many girls did x kiss?) $\rightsquigarrow \lambda n|(\lambda y[{}^*GIRL(y) \wedge {}^*KISS(x,y)])| \geq n$

T^1 = three $\rightsquigarrow \lambda P[P(3)] = \lambda P \exists n[n = 3 \wedge P(n)]$

T^2 = three $\rightsquigarrow \lambda n[n=3]$

$$\text{exh}(18') = \text{for every boy x, } \{|(\lambda y[{*}\text{GIRL}(y) \wedge {*}\text{KISS}(x,y)]| \geq 3 \wedge$$
$$\forall Q[[Q(3) \wedge Q \subseteq \lambda n|\lambda y({*}\text{GIRL}(y) \wedge {*}\text{KISS}(x,y)]| \geq n] \rightarrow$$
$$\max Q \leq \max(\lambda n[n=3] \cap \lambda n|\lambda y[{*}\text{GIRL}(y) \wedge {*}\text{KISS}(x,y)]| \geq n)]$$

In words: Every boy is such that, the number of girls that he kissed is at least 3, and for every subset of numbers of girls kissed by him, and which contains 3, its largest member is smaller than or equal to the largest number in the intersection of {3} and the set of numbers of girls that he kissed.

Exh(18') means that every boy kissed exactly 3 girls.

5.1.2 Beer and orange juice

As noted in chapter 1, section 1.7, 'embedded' implicatures show up also in clearly context dependent cases, such as example (21) below:

(21) A: Did everyone order beer?

B: Some ordered orange juice

We understand B's sentence as implicating that some did not order beer, hence the answer to A's question is "no". B's reply answers A's question indirectly. B chose not to answer A's question, but another question, namely "For some x, what did x order?". The answer to this question, if understood exhaustively, entails the answer to A's original question. I'll show how exhaustivity works in this case, using the formulation that was given in example (18') above.

(22) For some x, what did x order?

Someone ordered [orange juice]$_F$

P= ABS(What did x order?) $\rightsquigarrow \lambda y$*order(x,y)

T^1= orange juice $\rightsquigarrow \lambda P.P$(orange juice)

T^2=BE(orange juice) = λx(x=orange juice)

exh(21) = For some x, {*order(x,orange juice)$\wedge\forall$Q[[Q(orange juice)$\wedge$Q $\subseteq$ λy*order(x,y)] $\rightarrow \sigma Q \sqsubseteq$ orange juice]}

In words: For some x, x ordered orange juice, and for every subset of x's orders that includes orange juice, its sum is part of orange juice.

It's not hard to see that exh(22) means that there is someone who ordered only orange juice.

We see that the exhaustivity analysis deals quite easily with cases of context dependent 'embedded implicatures'. If the facts about implicatures in logically complex sentences suggest that implicatures are computed by grammar, the exhaustivity analysis is less stipulative than Landman's and Chierchia's theories. On their theories one would have to assume that an ad hoc Horn scale triggered in a certain context gives rise to a 'local implicature', that inherits up according to the semantic composition of the sentence. It is not clear to me why pragmatic considerations of 'informativeness' which are supposed to be the basis of ad hoc Horn scales such as (*orange juice*, *orange juice and beer*) should introduce 'local', rather than 'global' implicatures. The term *orange juice* is not a scalar term. The set of alternatives {orange juice, orange juice and beer} would makes sense here only after understanding that the reply given for the question would entail the answer only after accommodating the implicature. On the other hand, on the exhaustivity analysis, the inferences in question are merely the result of applying a semantic operator relating questions and answers. The context dependency of these inferences comes from the fact that we get different exhaustivizations relative to different questions. We get a 'global' exhaustivization, hence a 'global' implicature when the focused element has widest scope – it serves as a short answer to a 'global' question (examples 17 and 18 above). We get a 'local' exhaustivization, hence a 'local' implicature, when the focused element is embedded – it serves as a short answer to a 'local' question (examples 17', 18' and 22 above).

5.1.3 Embedding under intensional verbs

Let us consider now the case of an embedding verb.

(23) Who does John believe came?
Sue or Bill

P= ABS(who does John believe came?) $\rightsquigarrow \lambda x$BELIEVE(j, ^*COME(x))

T^1= sue or bill $\rightsquigarrow \lambda P[P(s) \vee P(b)]$

T^2 = sue or bill $\rightsquigarrow \lambda x[(x=s) \vee (x=b)]$

$$\text{Exh}(23) = [\text{BELIEVE}(j, {}^\wedge{*}\text{COME}(s)) \vee \text{BELIEVE}(j, {}^\wedge{*}\text{COME}(b))] \wedge$$

$$\forall Q[[[Q(s) \vee Q(b)] \wedge Q \subseteq \lambda x \text{BELIEVE}(j, {}^\wedge{*}\text{COME}(x))] \rightarrow$$

$$\sigma Q \sqsubseteq \sigma[[\lambda x[(x=j) \vee (x=m)] \cap \lambda x \text{BELIEVE}(j, {}^\wedge{*}\text{COME}(x))]]]$$

The first conjunct of exh(23) ensures that λxBELIEVE(j, ^*COME(x)) includes Sue or Bill, hence $\sigma[[\lambda x[(x=j) \vee (x=m)] \cap \lambda x$BELIEVE(j, ^*COME(x))]] is only defined if λxBELIEVE(j, ^*COME(x))= {s} or if λxBELIEVE(j, ^*COME(x)) = {b}, and exh(23) reduces to:

$$\text{Exh}(23) = [\text{BELIEVE}(j, {}^\wedge{*}\text{COME}(s)) \vee \text{BELIEVE}(j, {}^\wedge{*}\text{COME}(b))] \wedge \forall Q[[[Q(s) \vee Q(b)] \wedge Q \subseteq \lambda x \text{BELIEVE}(j, {}^\wedge{*}\text{COME}(x))] \rightarrow (\sigma Q \sqsubseteq s \vee \sigma Q \sqsubseteq b)]$$

In words: John believes that Sue came or John believes that Bill came, and for every subset of individuals who John believes came which includes Sue or which includes Bill, its sum is part of Sue or part of Bill.

It is not hard to see that exh(23) means that John believes that only Sue came or that only Bill came. We get the right facts about the exclusive implicature of *or* for this case.

Let us do now a case with de dicto/de re ambiguity. I start with the de re reading.

(24) Who does John believe came?

The dean

P= ABS(who does John believe came?) $\rightsquigarrow \lambda x BELIEVE(j, {}^\wedge{*}COME(x))$

T^1= the dean $\rightsquigarrow \lambda P[P(\sigma(DEAN)]$

T^2 = the dean $\rightsquigarrow \lambda x[x= \sigma(DEAN)]$

$$Exh(24) = [\lambda x BELIEVE(j, {}^\wedge{*}COME(x)](\sigma(DEAN)) \wedge$$
$$\forall Q[[[Q(\sigma(DEAN)] \ \wedge Q \subseteq \lambda x BELIEVE(j, {}^\wedge{*}COME(x))] \rightarrow$$
$$\sigma Q \sqsubseteq \sigma(\lambda x[x=\sigma(DEAN)] \cap \lambda x BELIEVE(j, {}^\wedge{*}COME(x))]$$

The first conjunct of exh(24) ensures that $\lambda x BELIEVE(j, \lambda w{*}COME(x,w))$ includes $\sigma(DEAN)$, hence $\sigma(\lambda x[x= \sigma(DEAN)] \cap \lambda x BELIEVE(j, \lambda w{*}COME(x,w))$ is only defined if $\lambda x BELIEVE(j, \lambda w{*}COME(x,w)) = \sigma(DEAN)$, and exh(24) reduces to:

$$Exh(24) = [\lambda x BELIEVE(j, \lambda w{*}COME(x,w))](\sigma(DEAN)) \wedge$$
$$\forall Q[[[Q(\sigma(DEAN)] \ \wedge Q \subseteq \lambda x BELIEVE(j, \lambda w{*}COME(x,w))] \rightarrow \sigma Q \sqsubseteq \sigma(DEAN)]$$

In words: The dean is such a person whom John believes that came, and for every subset of individuals whom John believes that came which includes the dean, its sum is part of the dean.

Exh(24) means that John believes of the dean that (s)he came, and that this person is the only person of whom John believes that (s)he came.

In the de dicto reading, *the dean* does not refer to the dean in the real world, but to the **intension** of *the dean*.

(25) Who does John believe came?

The dean

P= ABS(who does John believe came?) $\rightsquigarrow$

$\lambda s BELIEVE(j, {}^\wedge{*}COME({}^\vee s))$, where s is a variable of type <s,e>

T^1= the dean $\rightsquigarrow$ $\lambda P[P({}^\wedge\sigma(DEAN))]$, P is a variable of type <<s,e>,t>

T^2 = the dean $\rightsquigarrow \lambda s[s={}^\wedge\sigma(DEAN)]$

$T^1(P) = \lambda P[P({}^\wedge\sigma(DEAN))]\ [\lambda s BELIEVE(j, {}^\wedge{*}COME({}^\vee s))] =$

$[\lambda s BELIEVE(j, {}^\wedge{*}COME({}^\vee s))]({}^\wedge\sigma(DEAN)) = BELIEVE(j, {}^\wedge COME(\sigma(DEAN))$

$\sqsubseteq$ is lifted from type e to type <s,e> by:

$s \sqsubseteq t$ iff for every w: $s(w) \sqsubseteq t(w)$

$$Exh(25) = BELIEVE(j, {}^\wedge COME(\sigma(DEAN)) \wedge$$

$$\forall Q[[Q({}^\wedge\sigma(DEAN) \wedge Q \subseteq \lambda s BELIEVE(j, {}^\wedge{*}COME({}^\vee s))] \rightarrow$$

$$\sigma Q \sqsubseteq \sigma[\lambda s[s={}^\wedge\sigma(DEAN)] \cap \lambda s BELIEVE(j, {}^\wedge{*}COME({}^\vee s))]]$$

The first conjunct of exh(25) ensures that $\lambda s BELIEVE(j, {}^\wedge{*}COME({}^\vee s))$ includes ${}^\wedge\sigma(DEAN)$, hence $\lambda s[s={}^\wedge\sigma(DEAN)] \cap \lambda s BELIEVE(j, {}^\wedge{*}COME({}^\vee s))] =$ $\lambda s[s={}^\wedge\sigma(DEAN)]$, and $\sigma[\lambda s[s={}^\wedge\sigma(DEAN)] = {}^\wedge\sigma(DEAN)$, and exh(25) reduces to:

$$Exh(25) = BELIEVE(j, {}^\wedge COME(\sigma(DEAN)) \wedge$$

$$\forall Q[[Q({}^\wedge\sigma(DEAN) \wedge Q \subseteq \lambda s BELIEVE(j, {}^\wedge{*}COME({}^\vee s))] \rightarrow$$

$$\sigma Q \sqsubseteq {}^\wedge\sigma(DEAN)]$$

In words: John believes that the dean came, and for every subset of the set of individual concepts that includes the dean concept, such that every individual concept in that set is a function such that John believes that its value came, the sum of that set is part of the dean concept.

The only way to satisfy exh(25)'s truth conditions is by letting $\lambda s \text{BELIEVE}(j, {}^\wedge{*}\text{COME}({}^\vee s)) = \{{}^\wedge\sigma(\text{DEAN})\}$.

In the de dicto reading of *John believes [the dean]*$_F$ *came*, the focal element (*the dean)* truth conditionally seems to have narrow scope under the propositional attitude (*believe*). But unlike examples (17'), (18') and (22) above (where we got 'local' implicatures), the question here is 'global' (it is about an individual concept which takes widest scope), and we get a 'global' implicature – the dean is the only individual concept such that John believes its value came. Note, however, that it is also possible to get 'local' implicatures with de dicto readings. If, for example, we take *John believes that 3 men came* as an answer to the in situ interrogative *John believes for what n that n is a number of men who came?*, we would expect exhaustivization to take place under the scope of *believe*, and the exhaustive reading would be that John believes that exactly 3 men came.

5.2 Scalar implicatures under negation and in other downward entailing contexts

Gazdar (1979) notes that Scalar implicatures tend to be suspended under negation. Indeed, the prominent interpretation of sentence (26) below is that John has less than 3 children, and not that he either has less than 3 children or more than 3 children.

(26) John doesn't have 3 children

Gazdar limits the computation of implicatures to cases were the scalar trigger is not in the scope of another logical operator. We saw in the previous section that this is wrong. Implicatures do appear under the scope of quantifiers and embedding verbs. Hirschberg (1985) suggests that only overt negation blocks scalar implicatures. Horn (1989) comments that scalar implicatures are suspended not only under negation, but generally in downward entailing contexts, but doesn't give supporting evidence. Chierchia (ms) claims that scalar implicatures are suspended in the contexts that license *any* (as a negative polarity or as a free choice item). According to Chierchia, the exclusive interpretation of *or* is missing in all of the following cases (the examples are taken from Chierchia).

(27) **Negation**:

Sue didn't meet Hugo or Theo.

(28) **Negative Quantifiers**:

No student with an incomplete or a failing grade is in good standing.

No student who missed class will take the exam or contact the advisor.

(29) **Restriction of *every***:

Every student who wrote a squib or made a classroom presentation got extra credit.

(30) **Antecedents of conditionals**:

If Paul or Bill come, Mary will be upset.

(31) **Negative embedding predicates**:

John doubts/regrets/fears that Paul or Bill ate in that room.

(32) **Generic statements**:

A linguist or a philosopher doesn't give easily in.

(33) ***Before***:

John arrived before Paul or Bill.

(34) ***Without***:

John will come without pen or notepads.

(35) **Comparatives**:

Theo is taller than Bill or John.

(36) **Verbs of comparison**:

I prefer Theo to John or Bill.

(37) **Modality of permission**:

You may smoke or drink.

(38) **Questions**:

Did John or Paul arrive?

(39) **Imperatives**:

Get me Paul or Bill.

(40) **Irrealis mood**:

Ci sara qualcuno che sappia inglese o francese!

(I hope) there will be somebody who knows English or French.

I think that the generalization is wrong, and that scalar implicatures are possible in most, if not all of the above contexts, as demonstrated in the following examples:

(41) **Negation**:

John doesn't have three kids – he is not the one to stop at an odd number of kids.

(42) **Negative quantifiers**:

No boy kissed 3 girls (but some kissed 2, and some kissed 4).

(43) **Restriction of *every***:

Everyone who spent $333 will get a full refund (those who spent more won't).

(44) **Antecedents of conditionals**:

If Paul is good at mathematics he'll get a grade between 85 and 90, if he's excellent, he'll get over 95.

(45) **Negative embedding predicates**:

John doubts that Sue has 3 children… he tends to believe that she has either 2 or 4.

(46) **Generic statements**:

A pretty girl is likely to find a date, but a gorgeous girl isn't. Not many guys would dare asking her.

(47) ***Before***:

Mother serves the soup boiling hot. Most people eat it before it gets cold, but Fred usually makes the mistake of eating it before it gets warm.

(48) ***Without***:

I can manage without decent wine, but not without superb wine.

(49) **Comparatives**:

John is taller than most (but not all) guys.[1]

(50) **Verbs of comparison**:

I prefer warm weather, but I detest hot weather.

(51) **Modality of permission**:

You may bring a pet (but not two).

(52) **Questions**:

A: Does John have 3 children?

B: No, he has 4.

(53) **Imperatives**:

A: How many eggs do you need?

B: Get me 3 eggs (no more and no less).

Both Landman(2000) and Chierchia (ms.) impose on their implicature inheritance mechanisms some constraint that will block the implicatures in downward entailing contexts.

As mentioned before, Landman suggests that scalar implicatures are introduced locally, and inherit up following the semantic composition of the sentence. Landman assumes that the implicature won't inherit up if the implicature calculated at some

[1] The issues of downward entailingness and licensing of polarity items in comparatives are highly complex and problematic (See Schwarzschild & Wilkinson 2002).

stage contradicts the meaning calculated at that stage. Let us assume that in sentence (54) below, the scalar implicature is introduced at the level of *have 3 children*, and is the set (55).

(54) John doesn't have 3 children

(55) {x: x has no more than 3 children}

In the process of inheriting up, we need to apply negation, and will get (56):

(56) {x: x has more than 3 children}

Now, (55) contradicts the meaning built at this stage:

(57) {x: x has less than 3 children}

And the inheritance of the implicature would stop.

Chierchia assumes a constraint to the effect that a strengthened semantic value of a sentence is rejected if it is no stronger than the plain semantic value. The strengthened semantic value of *John doesn't have 3 children* would be *John doesn't have exactly 3 children*, which is weaker than its meaning, hence it is rejected.

It is important to note that neither Landman nor Chierchia can derive the correct implicature for the following case:

(58) A: Whom did no girl kiss?

B: No girl kissed [John or Bill]$_F$

Maybe the most natural interpretation of (58B) is equivalent to "No girl kissed John **and** No girl kissed Bill" (which results from an inclusive interpretation of *or*). But an exhaustive reading (with an exclusive interpretation of *or*) is also possible here, and B's utterance can be interpreted as conveying that either the only one which no girl kissed is John, or the only one which no girl kissed is Bill.

On Landman's theory, the exclusive implicature of *or* comes at the VP level, the implicature of {x: x kissed John or Bill} being {x: x didn't kiss both John and Bill} Applying this to *no girl* will result in *No girl is such that she didn't kiss both John and Bill*, i.e. *Every girl kissed John and Bill*. This clearly contradicts the meaning of the sentence, so this implicature is cancelled. Although Landman's theory predicts correctly that *No girl kissed John and Bill* does not implicate that every girl kissed John and Bill, it does not gives us a way to compute the correct implicature of this sentence.

On Chierchia's inheritance mechanism, we first compute the strengthened value of *kissed John or Bill* which is *kissed John or Bill and not both*. We apply this to *no girl* (we do not have to worry about the scalar element *no* because it is the highest value in its scale), and get *No girl kissed (John or Bill and not both)*. This means that every girl either kissed both John and Bill or none of them. This strengthened meaning is weaker than the sentence's meaning which is equivalent to 'every girl didn't kiss John and Bill', hence it is rejected. Chierchia's theory too succeeds in filtering out the wrong implicature, but does not have a way to predict the correct implicature.

Let us see now what are the predictions of the exhaustivity theory of scalar implicatures concerning these cases. According to our theory, the implicatures of a sentence depend crucially on the question that it answers. For example, the sentence *John has 3 children* will be interpreted as *John has exactly 3 children,* only if it is taken to answer the question *How many children does John have?*. The exactly effect won't show up if we take our sentence to answer the question *Who has 3 children?* or the question *does John have 3 children?* (ignoring, for now, the possibility that the question itself might carry the exactly implicature). Concerning the negated sentence, *John doesn't have 3 children,* the exactly implicature cannot show up if it answers *who doesn't have 3 children?* or *does John have 3 children?*. Our best bet to get an exactly implicature for this sentence is to interpret it as an answer to *How many children doesn't John have?*. I will show that in this case as well, an exactly reading does not exist, because exhaustivization yields a contradiction.

(59) How many children doesn't John have?

Three

P = ABS(how many children doesn't john have?) $\rightsquigarrow \lambda n|(\lambda y[{}^*\mathrm{CHILD}(y) \wedge$

$\neg {}^*\mathrm{HAVE}(j,y)])| \geq n$

T^1 = three $\rightsquigarrow \lambda P[P(3)] = \lambda P \exists n[n = 3 \wedge P(n)]$

T^2 = three $\rightsquigarrow \lambda n[n=3]$

$$exh(59) = |(\lambda y[*CHILD(y) \wedge\neg *HAVE(j,y)]| \geq 3 \wedge \forall Q[[Q(3) \wedge Q \subseteq \lambda n|\lambda y(*CHILD(y) \wedge\neg *HAVE(j,y)]| \geq n] \rightarrow maxQ \leq max(\lambda n[n=3] \cap \lambda n|\lambda y[*CHILD(y) \wedge\neg *HAVE(j,y)]| \geq n)]$$

In words: John doesn't have 3 children or more, and for every subset of numbers of children that John doesn't have and which contains 3, its largest member is smaller than or equal to the largest number in the intersection of {3} and the set of numbers of children that John doesn't have.

Exh(59)'s truth conditions cannot be fulfilled. The first main conjunct of exh(59) ensures that John doesn't have 3 children or more. This means that the set of numbers of children that John doesn't have is{3, 4…}. The second main conjunct of exh(59) requires that the largest number in every subset of this set is smaller than or equal to 3. Hence, The set of numbers of children that John doesn't have must be {3}. But this is not a valid set of numbers of children that John doesn't have. If John doesn't have 3 children, he also doesn't have 4 children or more. Exhaustivization in this case yields a contadiction, so I assume that we do not exhaustivize.

Indeed, 3 cannot be an exhaustive answer to the question *how many children doesn't John have?* An exhaustive answer in this case would be *3 or more*.

(60) How many children doesn't John have?

Three or more

P = ABS(how many children doesn't john have?) $\rightsquigarrow \lambda n|(\lambda y[*CHILD(y) \wedge$

$\neg$*HAVE(j,y)])$| \geq n$

T^1 = three or more $\rightsquigarrow \lambda P \exists n[n \geq 3 \wedge P(n)]$

T^2 = three $\rightsquigarrow \lambda n[n \geq 3]$

exh(60) = $|(\lambda y[*CHILD(y) \wedge \neg *HAVE(j,y)]| \geq 3 \wedge \forall Q[[Q(\geq 3) \wedge Q \subseteq \lambda n|\lambda y(*CHILD(y) \wedge \neg *HAVE(j,y)]| \geq n] \rightarrow maxQ \leq max(\lambda n[n \geq 3] \cap \lambda n|\lambda y[*CHILD(y) \wedge \neg *HAVE(j,y)]| \geq n)]$

In words: John doesn't have 3 children or more, and for every subset of numbers of children that John doesn't have and which contains 3 or a larger number, its largest member is smaller than or equal to the largest number in the intersection of {3,4,..} and the set of numbers of children that John doesn't have.

It's easy to see that in this case exhaustivization does not have any effect.

The fact that the exhaustivization of (59) is not possible should not come as surprise. The exhaustivization of *John has 3 children* in the context of *how many children does John have?* is equivalent to *The number of John's children is exactly 3*. Similarly, the exhaustivization of *John doesn't have 3 children* in the context of *How many children doesn't John have* is equivalent to *the number of children that John doesn't have is 3,* which is always undefined, because there is no such number.

I think, however, that sometimes we do apply *exh* under the scope of negation to yield an exactly reading (and hence there is no need to assume that these are cases of metalinguistic negation). Consider (59'):

(59') How many children does John have?

John doesn't have $[3]_F$ children (he is not the one to stop at an odd number).

Nirit Kadmon (p.c.) has suggested to me that the reply in (59') can be interpreted as conveying that 3 is not the exhaustive answer.

P = ABS(how many children does john have?) $\rightsquigarrow \lambda n|(\lambda y[*CHILD(y)\wedge *HAVE(j,y)])| \geq n$

T^1 = three $\rightsquigarrow \lambda P[P(3)] = \lambda P \exists n[n = 3 \wedge P(n)]$

T^2 = three $\rightsquigarrow \lambda n[n=3]$

exh(59') = it is not the case that: $\{|(\lambda y[*CHILD(y) \wedge *HAVE(j,y)]| \geq 3 \wedge$

$\forall Q[[Q(3) \wedge Q \subseteq \lambda n|\lambda y(*CHILD(y) \wedge *HAVE(j,y)]| \geq n] \rightarrow$

$maxQ \leq max(\lambda n[n=3] \cap \lambda n|\lambda y[*CHILD(y) \wedge \neg *HAVE(j,y)]| \geq n)]\}$

In words: It is not the case that: John has 3 children or more, and for every subset of numbers of children that John has and which contains 3, its largest member is smaller than or equal to the largest number in the intersection of {3} and the set of numbers of children that John has.

Exh(59') means that it is not the case that John has exactly 3 children (he may have less than 3 or more than 3). This is a case where the exhaustive reading contradicts the non exhaustive reading, and maybe this is the reason why we use the special 'meta-linguistic' intonation. This, however, shouldn't keep *exh* from applying, as the contradiciton is not within the result of *exh*. In this case the implicature is local – it falls within the scope of negation.

I will show now that exhaustivization predicts the correct implicature for (58), which is repeated as (61) below.

(61) Whom did no girl kiss?

No girl kissed $[\text{John or Bill}]_F$

P = ABS(whom did no girl kiss?) $\rightsquigarrow \lambda y[\text{GIRL} \cap \lambda x{*}\text{KISS}(x,y)=\varnothing]$

T^1 = john or bill $\rightsquigarrow \lambda P[P(j) \vee P(b)]$

T^2 = john or bill $\rightsquigarrow \lambda x[x=j \vee x=b]$

$$\text{exh}(61) = [\text{GIRL}\cap\lambda x{*}\text{KISS}(x,j) = \varnothing \vee \text{GIRL}\cap\lambda x{*}\text{KISS}(x,b) = \varnothing] \wedge$$
$$\forall Q[[[Q(j) \vee Q(b)] \wedge Q \subseteq \lambda y[\text{GIRL}\cap\lambda x{*}\text{KISS}(x,y)=\varnothing]] \rightarrow$$
$$\sigma Q \sqsubseteq \sigma[[\lambda x[(x=j)\vee(x=b)] \cap \lambda y[\text{GIRL}\cap\lambda x{*}\text{KISS}(x,y)=\varnothing]]]$$

The first conjunct of exh(61) ensures that $\lambda y[\text{GIRL}\cap\lambda x{*}\text{KISS}(x,y)=\varnothing]$ includes at least John or that it includes at least Bill, hence $\sigma[[\lambda x[(x=j)\vee(x=b)] \cap \lambda y[\text{GIRL}\cap\lambda x{*}\text{KISS}(x,y)=\varnothing]]$ is only defined if $\lambda y[\text{GIRL}\cap\lambda x{*}\text{KISS}(x,y)=\varnothing]= \{j\}$ or if $\lambda y[\text{GIRL}\cap\lambda x{*}\text{KISS}(x,y)=\varnothing]= \{b\}$, and exh(46) reduces to:

$$Exh(61) = [GIRL \cap \lambda x {*}KISS(x,j) = \varnothing \vee GIRL \cap \lambda x {*}KISS(x,b) = \varnothing] \wedge$$
$$\forall Q[[[Q(j) \vee Q(b)] \wedge Q \subseteq \lambda y[GIRL \cap \lambda x {*}KISS(x,y)=\varnothing]] \rightarrow$$
$$\sigma Q \sqsubseteq (\sigma Q \sqsubseteq j \vee \sigma Q \sqsubseteq b)]$$

In words: No girl kissed John or no girl kissed Bill and for every subset of the individuals who no girl kissed and which includes John or which includes Bill, its sum is part of John or part of Bill.

So, exh(61) means that either the only individual which no girl kissed is John or the only individual which no girl kissed is Bill. This interpretation is certainly available in this context, and the fact that the exhaustivity theory predicts it is an important advantage over Landman's and Chierchia's theories.

I will now do three examples with *doubt*.

(62) Who does John doubt came?

Sue or Bill

P= ABS(who does John doubt came?) $\rightsquigarrow \lambda x DOUBT(j, {}^\wedge{*}COME(x))$

T^1= sue or bill $\rightsquigarrow \lambda P[P(s) \vee P(b)]$

T^2 = sue or bill $\rightsquigarrow \lambda x[(x=s) \vee (x=b)]$

$$Exh(62) = [DOUBT(j, {}^\wedge{*}COME(s)) \vee DOUBT(j, {}^\wedge{*}COME(b))] \wedge$$
$$\forall Q[[[Q(s) \vee Q(b)] \wedge Q \subseteq \lambda x DOUBT(j, {}^\wedge{*}COME(x))] \rightarrow$$
$$\sigma Q \sqsubseteq \sigma[[\lambda x[(x=j) \vee (x=m)] \cap \lambda x DOUBT(j, {}^\wedge{*}COME(x))]]]$$

The first conjunct of exh(62) ensures that λxDOUBT(j, ^*COME(x)) includes Sue or Bill, hence $\sigma[[\lambda x[(x=j)\vee(x=m)] \cap \lambda x$DOUBT(j, ^*COME(x))]] is only defined if λxDOUBT(j, ^*COME(x))= {s} or if λxDOUBT(j, ^*COME(x)) = {b}, and exh(62) reduces to:

$$\text{Exh(62)} = [\text{DOUBT}(j, {}^\wedge{*}\text{COME}(s))\vee\text{DOUBT}(j, {}^\wedge{*}\text{COME}(b))] \wedge$$
$$\forall Q[[[Q(s)\vee Q(b)] \wedge Q \subseteq \lambda x\text{DOUBT}(j, {}^\wedge{*}\text{COME}(x))] \rightarrow (\sigma Q \sqsubseteq s \vee \sigma Q \sqsubseteq b)]$$

In words: John doubts that Sue came or John doubts that Bill came, and for every subset of individuals who John doubts came which includes Sue or which includes Bill, its sum is part of Sue or part of Bill.

Exh(62) means that John only doubts that SUE came or John only doubts that BILL came. The exhaustivity theory predicts correctly that such an exclusive interpretation is possible.

(63) How many girls does John doubt Bill kissed?
John doubts that Bill kissed 3 girls

First I’ll do the reading in which *3* takes wide scope relative to *doubt* (i.e. the question is interpreted as asking what is a number *n* such that there are *n* girls that John doubts Bill kissed).

P = ABS(How many girls does John doubt Bill kissed?) $\rightsquigarrow$

$\lambda n|(\lambda y[{*}\text{GIRL}(y) \wedge \text{DOUBT}(j, {}^\wedge{*}\text{KISS}(b,y))]| \geq n$

T^1 = three $\rightsquigarrow \lambda P[P(3)] = \lambda P \exists n[n = 3 \wedge P(n)]$

T^2 = three $\rightsquigarrow \lambda n[n=3]$

$$\text{exh}(63) = |(\lambda y[*\text{GIRL}(y) \wedge \text{DOUBT}(j, \wedge{*}\text{KISS}(b,y))]| \geq 3 \wedge$$
$$\forall Q[[Q(3) \wedge Q \subseteq \lambda n|\lambda y(*\text{GIRL}(y) \wedge \text{DOUBT}(j, \wedge{*}\text{KISS}(b,y))]| \geq n] \rightarrow$$
$$\max Q \leq \max(\lambda n[n=3] \cap \lambda n|\lambda y[*\text{GIRL}(y) \wedge \text{DOUBT}(j, \wedge{*}\text{KISS}(b,y))]| \geq n)]$$

In words: There are at least 3 girls of whom John doubts that Bill kissed, and for every subset of numbers of girls of whom John doubts that Bill kissed, and which contains 3, its largest member is smaller than or equal to the largest number in the intersection of {3} and the set of numbers of girls of whom John doubts that Bill kissed.

Exh(63) means that there are exactly 3 girls of whom John doubts that Bill kissed. Exh(63)'s first main conjunct requires that there are at least 3 girls of whom John doubts Bill kissed. I.e. the set of numbers of girls of whom John doubts that Bill kissed is {1,2,3} or {1,2,3,4} or {1,2,3,4,5}… The second main conjunct of exh(63), requires that the maximal number of every subset of this set which contains 3, is smaller or equal to 3. I.e. the set is {1,2,3}. The exhaustivity theory predicts correctly that this reading of *John doubts that Bill kissed 3 girls* has the exactly implicature in this context.

(63) has also a reading in which *3* takes narrow scope relative to *doubt.* In this reading, (63) can be taken as an answer to *what is a number n such that John doubts that n is a number of girls that Bill kissed*?

(63’) How many girls does John doubt Bill kissed?

John doubts that Bill kissed 3 girls

P = ABS(How many girls does John doubt Bill kissed?) $\rightsquigarrow$

$\lambda n DOUBT(j, {}^\wedge[|{*}GIRL(y) \wedge \lambda y{*}KISS(b,y)| \geq n])$

T^1 = three $\rightsquigarrow \lambda P[P(3)] = \lambda P \exists n[n = 3 \wedge P(n)]$

T^2 = three $\rightsquigarrow \lambda n[n=3]$

exh(63’) = $DOUBT(j, {}^\wedge[|{*}GIRL(y) \wedge {*}KISS(b,y)| \geq 3]) \wedge$

$\forall Q[[Q(3) \wedge Q \subseteq \lambda n DOUBT(j, {}^\wedge[|{*}GIRL(y) \wedge \lambda y{*}KISS(b,y)| \geq n]) \rightarrow$

$maxQ \leq max(\lambda n[n=3] \cap \lambda n DOUBT(j, {}^\wedge[|{*}GIRL(y) \wedge \lambda y{*}KISS(b,y)| \geq n])]$

In words: John doubts that Bill kissed at least 3 girls, and for every subset of numbers such that John doubts that Bill kissed at least that many girls, and which contains 3, its largest member is smaller than or equal to the largest number in the intersection of {3} and the set of numbers such that John doubts that Bill kissed at least that many girls.

Exh(63’) cannot be true. The reason is the same as for example (59). The first main conjunct of exh(63’) requires that John doubts that Bill kissed at least 3 girls. But if John doubts that Bill kissed 3 girls, then he also doubts that Bill kissed 4 girls etc… Hence the total set of such numbers is {3,4,5,…}, and this set doesn’t have a maximal element. The exhaustivity theory predicts correctly that this reading of *John doubts that Bill kissed 3 girls* cannot have the exactly implicature in this context.

Chierchia gives the following example from Levinson(2000) to show that sometimes scalar implicatures do show up in downward entailing environments.

(64) If John has two cars, the third one parked outside must be somebody else's.

Chierchia concludes that this implicature is accommodated somehow to the antecedent of the conditional, because it cannot be inherited up by his mechanism. He assumes that in this example we restrict our consideration to sets of worlds from which people with more than two cars are excluded. In the exhaustivity theory we do not have to assume a special accommodation procedure for such cases.

(65) A: How many cars does John have, such that if he has that many cars, the third one parked outside must be somebody else's?

B: If John has $[\text{two}]_F$ cars, the third one parked outside must be somebody else's?

I think that A's question should be interpreted along the lines of the slightly awkwardly formulated in situ question *If the answer to "how many cars does J have?" is* ***what****, then the third one parked outside must be somebody else's?* I.e. I assume that the interrogative sentence here is a conditional which embeds a question in its antecedent. The exhaustivization will take place inside the antecedent.

P = ABS(If the answer to "how many cars does J have?" is what, then the third one parked outside must be somebody else's?) $\rightsquigarrow$ If $\lambda n|(\lambda y[*CAR(y) \wedge *HAVE(j,y,])| \geq n$, the third one parked outside must be somebody else's

T^1 = two $\rightsquigarrow \lambda P[P(2)] = \lambda P \exists n[n = 2 \wedge P(n)]$

T^2 = two $\rightsquigarrow \lambda n[n=2]$

$$\begin{aligned}
exh(65) = \text{if } \{&|(\lambda y[*CAR(y) \wedge *HAVE(j,y)]| \geq 2 \wedge \\
&\quad \forall Q[[Q(2) \wedge Q \subseteq \lambda n|\lambda y(*CAR(y) \wedge *HAVE(j,y)]| \geq n] \rightarrow \\
&\qquad maxQ \leq max(\lambda n[n=2] \cap \lambda n|\lambda y[*CAR(y) \wedge *HAVE(j,y)]| \geq n)]\},
\end{aligned}$$

the third one parked outside must be somebody else's.

In words: If {John has at least 2 cars, and for every subset of numbers of cars owned by John which contains 2, its largest member is smaller than or equal to the largest number in the intersection of {2} and the set of numbers of cars owned by John}, the third one parked outside must be somebody else's.

Exh(65) means that if John has exactly 2 cars, the third one parked outside must be somebody else's.

5.3 Suspension or cancellation of scalar implicatures in non downward entailing environments

Consider the examples given in (66) and (67) below:

(66) a. Three, if not four men came

b. Three and maybe four men came

c. Four, or at least three men came

(67) Three, actually/in fact, four men came

Horn (1989) calls the environments in (66) **suspenders**: the speaker is explicitly leaving the possibility open that a higher value on the relevant scale obtains. The environments in (67), according to Horn, do not just suspend but **cancel** the scalar implicature, they explicitly assert that a higher value on the scale obtains.

Examples such as in (66a) and (66b) are discussed in Gazdar (1979). According to Gazdar, elements on Horn scales introduce potential scalar implicatures, which will turn to actual implicatures only under certain conditions. Gazdar claims that on the utterance of U, first the entailments of U are added to the context. Next, all the potential clausal implicatures are added that are consistent with the context (which includes now the entailments of U). Finally, potential scalar implicatures will be added, only if they are consistent with the context (which includes now also the clausal implicatures). On Gazdar's account potential scalar implicatures will turn to be actual implicatures only if they are consistent with the entailments and the clausal implicatures of U.

In sentence (66a) above, the potential scalar implicature, introduced by *3*, *no more than 3 men came*, is not realized as an actual implicature, because it contradicts the clausal implicature of the conditional, *It is possible that 4 men came*. In sentence

(66b), the potential scalar implicature, *John has no more than 4 children,* is not realized, because it contradicts the entailment *maybe 4 men* came.

Gazdar does not explain why clausal implicatures take precedence over scalar implicatures, hence is theory is somewhat arbitrary.

How should we analyze the cases in (66)-(67) within the exhaustivity theory of scalar implicatures?

I would like to claim that in none of the above cases an implicature is suspended or cancelled. The sentences in (66) are simply such that their exhaustivizations mean that exactly 3 or exactly 4 men come, whereas the exhaustivizations of the sentences in (67) mean that exactly 4 men came.

Winter (1998) mentions that the semantic effect of the modal *maybe* in sentence (68) is similar to a disjunction: (68) seems to be equivalent to (69).

(68) The guests are John, Bill, and Henry, and maybe Susan

(69) The guests are john, Bill, and Henry or John, Bill, Henry and Susan

Landman(2004) gives a compositional account of sentence adverbials inside noun phrase constructions. According to Landman, (68) asserts that the function which maps each epistemic alternative onto what the guests are according to that alternative is a function which maps alternatives either onto the sum of John, Bill and Henry, or onto the sum of John, Bill, Henry and Susan. We assert (68) when we don't yet know what the actual set of guests is, but we have reduced the alternatives to alternatives of the above two kinds. It seems that the same happens in (66b). *Three and maybe four* is equivalent to *three or four*.

Sentences (68) and (69) have equivalents with *if not* or *or at least* instead of *maybe* or *or*:

(70) The guests are John, Bill and Henry, if not John, Bill, Henry and Susan

(71) The guests are John, Bill, Henry and Susan, or at least John, Bill and Henry

So, the expressions *three if not four* in (66a) and *three or at least 4* in (66c) have also the interpretation *three or four*.

Now let us see what the exhaustivization of *three or four men came* (or *three and maybe four* or *if not three, four* or *three or at least four*) is in the context of *How many men came?*

(72) How many men came?

3 or 4 / 3 and maybe 4/ 3, if not4/ 3 or at least 4

P = ABS(how many men came?) $\rightsquigarrow \lambda n|(\lambda x[{*}MAN(x) \wedge {*}COME(x)])| \geq n$

T^1 = 3 or 4 /3 and maybe 4/3, if not4/3 or at least 4$\rightsquigarrow$

$\lambda P[P(3) \vee P(4)] = \lambda P\exists n[(n = 3 \vee n{=}4) \wedge P(n)]$

T^2 = three or four/ three and maybe four/three, if not four $\rightsquigarrow \lambda n[n{=}3 \vee n{=}4]$

$$exh(72) = [|(\lambda x[*MAN(x) \wedge *COME(x)]| \geq 3 \vee |(\lambda x[*MAN(x) \wedge *COME(x)]| \geq 4] \wedge$$

$$\forall Q[[[Q(3) \vee Q(4)] \wedge Q \subseteq \lambda n|\lambda x(*MAN(x) \wedge *COME(x)]| \geq n] \rightarrow$$

$$maxQ \leq max(\lambda n[n=3 \vee n=4] \cap \lambda n|\lambda y[*MAN(x) \wedge *COME(x)]| \geq n)]$$

In words: 3 men came or 4 men came, and for every subset of numbers of men who came which contains 3 or which contains 4, its largest member is smaller than or equal to the largest number in the intersection of {3,4} and the set of numbers of men who came.

Exh(72) means simply that either exactly 3 men came or exactly 4 men came.

Concerning (67), *Three, actually/in fact, four men came*, I assume that the interpretation of the expressions *three, actually four*, and *three, in fact four* is simply 4. One of the uses of *Actually*, and *in fact* is to state that the hearer should replace the information given in the preceding expression with the information given in the following. A constraint on this use is that the meaning of the 'correct' expression should be close to the meaning of the 'cancelled' expression. Compare (73) with the oddness of (74):

(73) The flat is painted white, actually/in fact, light beige

(74) #The flat is painted white, actually/in fact, red

Now, we are ready to analyze the cases in (67):

(75) How many men came?

Three, actually/in fact four

P = ABS(how many men came?) $\rightsquigarrow \lambda n|(\lambda x[{}^{*}MAN(x) \wedge {}^{*}COME(x)])| \geq n$

T^1 = three, actually/ in fact four $\rightsquigarrow \lambda P \exists n[(n = 4) \wedge P(n)]$

T^2 = three, actually/ in fact four $\rightsquigarrow \lambda n[n = 4]$

$$exh(75) = |(\lambda x[{}^{*}MAN(x) \wedge {}^{*}COME(x)]| \geq 4 \wedge$$
$$\forall Q[[Q(4) \wedge Q \subseteq \lambda n|\lambda x({}^{*}MAN(x) \wedge {}^{*}COME(x)]| \geq n] \rightarrow$$
$$maxQ \leq max(\lambda n[n{=}3 \sqcup 4] \cap \lambda n|\lambda y[{}^{*}MAN(x) \wedge {}^{*}COME(x)]| \geq n)]$$

In words: 4 men came, and for every subset of numbers of men who came which contains 4, its largest member is smaller than or equal to the largest number in the intersection of {4} and the set of numbers of men who came.

Exh (75) means that exactly 4 men came.

Real cases of suspending or canceling an implicature are cases where the implicature was indeed inferred, but then removed from some reason, like the cases in (76) – (77).

(76) A: How many men came?

B: Three. No/ in fact/ actually, four

(77) A: How many men came?

B: Three… Maybe four.

In (76), A accepts the answer *three*, and exhaustivizes it, i.e. interprets it as conveying that exactly 3 men came. When B corrects herself, A discards the answer *three*, accepts the new answer, *four*, and exhaustivizes it, i.e. interprets it as conveying that exactly 4 men came.

In (77), the procedure is a bit different. Initially, A accepts *three* as the answer, and exhaustivizes it, i.e. interprets it as conveying that exactly 3 men came. When B continues, A realizes that the exhaustive meaning is not compatible with the new information, she turns back to the non-exhaustive interpretation, adds to it the new information, and exhaustivizes the result, interpreting B's complete utterance as conveying that either exactly 3 or exactly 4 men came.

Chapter 6

Weak Quantity Implicatures

A weak quantity implicature of a sentence φ is the inference that *the speaker doesn't know/believe that ψ*, where ψ is a stronger statement than φ. In chapter 1, I showed that Gricean theories derive directly only weak quantity implicatures. These implicatures are often too weak. In order to strengthen a weak quantity implicature to a strong quantity implicature of the form *the speaker knows/believes that not ψ*, we need the additional premise that the speaker knows whether ψ. I discussed the fact that strong implicatures do exist in cases where this additional premise cannot be made (see discussion on chapter 1, sections 1.2, 1.3, 1.5 and 1.6). The exhaustivity analysis of implicatures (chapters 3 and 4) computes inferences which are not embedded under an epistemic operator (unless it is present in the sentence itself). So our "implicatures" are not generally of the form *the speaker doesn't know/believe than ψ*, or *the speaker knows/believes that not ψ*. But, if our theory predicts that a certain sentence in a certain context has the inference χ, Grice's maxim of Quality will ensure it has also the inference *the speaker knows/believes that χ*. Therefore, loosely speaking, the exhaustivity theory of implicatures computes only strong quantity implicatures. However, in some cases, such as the clausal implicatures of conditionals or disjunctions, or Grice's 'holiday in France' example (see discussion in chapter 1, section 1.2), all we get is the weak quantity implicature. The 'corresponding' strong implicature is not present. In the present chapter, I show how the exhaustivity operator helps us to derive these weak inferences.

6.1 Clausal implicatures

A kind of context dependent inferences that conditionals and disjunctions have, and which are labeled 'clausal implicatures', are exemplified in (1) and (2) below:

(1) If there's light in John's window, he is at home

Clausal implicatures: (i) As far as the speaker knows, it is possible that there is light in John's window

(ii) As far as the speaker knows, it is possible that there is no light in John's window

(iii) As far as the speaker knows ,it is possible that John is at home

(iv) As far as the speaker knows, it is possible that John isn't at home

(2) John or Bill came

Clausal implicatures: (i) As far as the speaker knows, it is possible that John came

(ii) As far as the speaker knows, it is possible that John didn't come

(iii) As far as the speaker knows, it is possible that Bill came

(iv) As far as the speaker knows, it is possible that Bill didn't come

On Gricean theories, the derivation of these implicatures is straightforward. Stronger alternatives to (2) are:

(3) John came

(4) Bill came

And (5)-(6) are derived as quantity implicatures:

(5) The speaker doesn't know that John came

(6) The speaker doesn't know that Bill came

In chapter 1, section 1.2, I argued that the epistemic operator used in the Gricean quantity implicatures is not *know* on its strong factive sense, but a weaker epistemic operator, something like 'follows from the available information'.

(5) is compatible with cases where that speaker does not know whether John came, and it is also compatible with cases where the speaker knows that John didn't come. But if the latter case were true, according to Quantity, the speaker would utter the more informative statement *Bill came*, and not the weaker statement *John or Bill*

came. Hence, the speaker doesn't know whether John came. Using similar considerations, from (6), we get that the speaker doesn't know whether Bill came.

The conditional case works much in the same way. If we take the conditional to be a strict implication, $\Box(\varphi\rightarrow\psi)$, we need to assume that the modal base of the necessity operator is the same as the modal base of the epistemic operator of the implicatures.

Now, I have argued in the bulk of this thesis, that we do not need the Quantity maxim for deriving scalar implicatures. If can show that weak quantity implicatures too can be given an analysis which does not refer to the Quantity maxim, we will no longer have use for the maxim at all, and we will be able to get rid of it altogether.

We have already analyzed in chapter 3, sentences such as in (7). The exhaustivization of the answer in (7) is given in (8).

(7) A: Who came?

B: John, Bill and Sue came

(8) Only John, Bill and Sue came

After exhaustivization, B fully answers A's question. (8) specifies all comers. By the maxim of **Quality** (which we do not get rid of), we can infer that B knows who came, and who didn't come. No weak implicature is derived in this case.

Now we look at the question/answer pair in (9). The exhaustivization of (9) is given in (10).

(9) A: Who came?

B: John or Bill

(10) Only John or only Bill came

Even after exhaustivization, B does not give a complete answer. We cannot infer the set of comers from B's answer. There are two possibilities here: either B knows the answer to the question *who came?*, and does not want to give it for some reason or another, or he does not know the answer to that question. Let us assume the latter option. By **Quality**, we can infer that B knows that either only John came, or that only Bill came. This allows 3 possibilities: B knows that only John came, B knows that only Bill came, B knows that only one of the two came, but he doesn't know which one. Only the third possibility is compatible with the assumption that B does not know the answer to *who came?* Therefore, we can derive the clausal implicatures that as far as B knows, it is possible that John came, it is possible that John didn't come, it is possible that Bill came, and it is possible that Bill didn't come.

In other words: Groenendijk and Stokhof's theory of questions defines the notion of an answer as a **complete** answer. John or Bill doesn't count as an answer to the question *who came?*, and from that and **Quality** we derive that B doesn't know the answer. So what B does instead, is give a partial answer. He does not answer A's question, but brings him closer to an answer.

We can stipulate, then, a maxim of **Quality** for question answering, which says: **Answer the Question!** This reinterprets the bit of **Quantity** used in explaining clausal implicatures in terms of **Quality**.

If we have a reason to think that B knows the full answer, but does not want to give it, the clausal implicatures would not be derived.

Let us do another example with a disjunction, but this time in the context of a yes/no question.

(11) A: Did John or Bill come?

B: Yes

I take it that a yes/no question denotes the set of worlds in which the answer to it is *yes*. The terms *yes* and *no*, are functions from sets of worlds to truth values. *Yes* maps a set of worlds to 1, if the world of evaluation is a member of the denotation of the question, and to 0 otherwise. We can think of *yes* and *no* as generalized quantifiers over sets of worlds. Let us see if and how we can use our formulation of exhaustivity in this case.

(12) Let P, Q be variables ranging over sets of worlds, partially ordered by ≤., s.t. <Q, ≤ > is a join semilattice.

We associate with *yes* and *no* two interpretations: an interpretation of type <<s,t>,t> and an interpretation of type <s,t>.

Let T be a variable of type <<s,t>,t>×<s,t> (a variable over pairs of sets of sets and sets). If $\alpha \in EXP_{<<e,t>,t>\times<e,t>}$ and $[\![\alpha]\!] = <\mathcal{T},\mathcal{P}>$, then $[\![\alpha^1]\!] = \mathcal{T}$ and $[\![\alpha^2]\!] = \mathcal{P}$.

$$exh = \lambda T\lambda P[T^1(P)\wedge\forall Q[[T^1(Q)\wedge Q \subseteq P] \rightarrow maxQ \leq max(T^2\cap P)]$$

The variable P (the question) in this case is a variable over sets of worlds. Q is also a variable over sets of worlds. T^1 (the 'generalized quantifier' interpretation of *yes* or *no*) is a variable over functions from sets of worlds to truth values. In order for all terms in the formula to be well defined, the variable T^2 must be a variable over sets of worlds. T^2 is the set interpretation of *yes* and *no*. The set interpretation of *yes* will be the set of worlds which are identical to the world of evaluation, i.e. the singleton set which includes only the world of evaluation.

Now I get to the interpretation of the partial order and the maximality operator. Sets of worlds, unlike plurality constructions and sets of cardinalities, do not normally have maximal elements (Unless we're dealing with phenomena such as counterfactuals that make use of a relation of similarity to the world of evaluation). I'll make a stipulation here. The set of possible worlds contains a special element, the world of evaluation, w_0. Let V be a set of worlds. $maxV = w_0$, if $w_0 \in V$, undefined otherwise. The partial ordering on the set of worlds will be defined as follows: for every world, w, $w \leq w_0$. Except of the world of evaluation, no world is ordered relative to another.

It is easy to see that any subset V of W which contains w_0 is a join semilattice. Any set of worlds which does not contain w_0 is not a join semilattice.

(11) A: Did John or Bill come?

B: Yes

P= ABS(did john or bill come?) $\rightsquigarrow \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)]$

T^1= Yes $\rightsquigarrow \lambda P[w_0 \in P]$; P is a variable of type <s,t>

T^2= $\text{BE}(T^1) = \lambda T \lambda w[T(\lambda v[v=w])](\lambda P[w_0 \in P]) = \lambda w[\lambda P[w_0 \in P](\lambda v[v=w])] =$

$\lambda w[w_0 \in (\lambda v[v=w])] = \lambda w[w_0 \in \{w\}] = \{w_0\}$

$$\begin{aligned}\text{Exh}(11) = {} & w_0 \in \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)] \wedge \\ & \forall Q[[w_0 \in Q \wedge Q \subseteq \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)]] \rightarrow \\ & \qquad \max Q \leq \max(\{w_0\} \cap \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)])]]\end{aligned}$$

The first main conjunct of Exh(11) states that $w_0 \in \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)]$, hence $\max(\{w_0\} \cap \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)])]] = w_0$. For every subset of worlds Q, which includes w_0, $\max Q = w_0$, and exh(11) reduces to:

$$\begin{aligned}\text{Exh}(11) = {} & w_0 \in \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)] \wedge \\ & \forall Q[[w_0 \in Q \wedge Q \subseteq \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)]] \rightarrow w_0 \leq w_0\end{aligned}$$

But $w_0 \leq w_0$ always holds, so the whole second main conjunct of exh(11) is trivial, and exh(11), reduced to:

$\text{Exh}(11) = w_0 \in \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)]$

So, exh(11) simply means that John or Bill come. We see that exhaustivization does not have any effect in this case. *Yes* is a full answer to a yes/no question. By Quality, we derive that B knows that John or Bill came, and no clausal implicatures will be derived, because we cannot make further assumptions about what B knows or doesn't know (B gave a full answer, hence B knows it, and that's it).

Let us do the same example, but with the answer *No*.

(13) A: Did John or Bill come?

B: No

P= ABS(did john or bill come?) $\rightsquigarrow \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)]$

T^1= No $\rightsquigarrow \lambda P[w_0 \notin P]$; P is a variable of type <s,t>

T^2= $\text{BE}(T^1) = \lambda T \lambda w[T(\lambda v[v=w])](\lambda P[w_0 \notin P]) = \lambda w[\lambda P[w_0 \notin P](\lambda v[v=w])] = \lambda w[w_0 \notin (\lambda v[v=w])] = \lambda w[w_0 \notin \{w\}] = W-\{w_0\}$

$$\begin{aligned}\text{Exh}(13) = {} & w_0 \notin \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)] \wedge \\ & \forall Q[[w_0 \notin Q \wedge Q \subseteq \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)]] \rightarrow \\ & \quad \max Q \leq \max((W-\{w_0\}) \cap \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)])]]\end{aligned}$$

The first main conjunct of Exh(13) states that $w_0 \notin \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)]$. All subsets Q of W which don't include w_0 are not join semilattices. Hence the second main conjunct of exh(13) is trivially true, and exh(13) reduces to:

$\text{Exh}(13) = w_0 \notin \lambda w[\text{COME}(j,w) \vee \text{COME}(b,w)]$

So, exh(13) simply means that it is not he case that John or Bill come. We see that exhaustivization does not have any effect in this case either. *No* is a full answer to a yes/no question, and no clausal implicatures will be derived.

I end this section with a re-discussion of the conditional example (26) from chapter 2, section 2.3, which is repeated as (14) below. First I want to show that as Groenendijk and Stokhof's formulation, our reformulation of exhaustivity also captures the implicature which strengthens a conditional to a bi-conditional.

(14) A: Does John come?

B: If Mary comes.

$P = \text{ABS(does john come?)} \rightsquigarrow \lambda w\text{COME}(j,w)$

$T^1 = \text{ if mary comes} \rightsquigarrow \lambda P[\lambda w\text{COME}(m,w) \subseteq P]$

$T^2 = \text{BE}(T^1) = \lambda T \lambda w[T(\lambda v[v=w])](\lambda P[\lambda u\text{COME}(m,u) \subseteq P]) =$

$\lambda w[(\lambda P[\lambda w\text{COME}(m,w) \subseteq P])\ (\lambda v[v=w])] = \lambda w[\lambda u\text{COME}(m,u) \subseteq (\lambda v[v=w])] =$

$\lambda w[\lambda u\text{COME}(m,u) \subseteq \{w\}] = \lambda w[\lambda u\text{COME}(m,u) \subseteq \{w\}] = \lambda w\text{COME}(m,w)$

$$\begin{aligned}\text{Exh}(14) = {} & [\lambda w\text{COME}(m,w) \subseteq \lambda w\text{COME}(j,w)] \wedge \\ & \forall Q[[[\lambda w\text{COME}(m,w) \subseteq Q] \wedge Q \subseteq \lambda w\text{COME}(j,w)] \\ & \qquad \rightarrow \max Q \leq \max(\lambda w\text{COME}(m,w) \cap \lambda w\text{COME}(j,w))]\end{aligned}$$

The first main conjunct of exh(14) states that [λwCOME(m,w) $\subseteq$ λwCOME(j,w)], hence (λwCOME(m,w) $\cap$ λwCOME(j,w)) = λwCOME(m,w), and exh(14) reduces to:

$$\text{Exh}(14) = [\lambda w\text{COME}(m,w) \subseteq \lambda w\text{COME}(j,w)] \wedge$$
$$\forall Q[[[\lambda w\text{COME}(m,w) \subseteq Q] \wedge Q \subseteq \lambda w\text{COME}(j,w)] \rightarrow$$
$$\max Q \leq \max(\lambda w\text{COME}(m,w))]$$

Let us assume that λwCOME(m,w) $\subseteq$ λwCOME(j,w) holds (i.e. if Mary comes, John comes), and let us distinguish 3 possible cases:

<u>First case</u>:

$w_0 \in \lambda$wCOME(m,w) and $w_0 \in \lambda$wCOME(j,w)

max(λwCOME(m,w)) = w_0

We look at every subset of worlds Q which is a both a subset of λwCOME(j,w) and a superset of λwCOME(m,w). Because $w_0 \in \lambda$wCOME(j,w) and $w_0 \in \lambda$wCOME(j,w), for every Q, $w_0 \in$Q, and for every Q, maxQ=w_0. $w_0 \leq w_0$, and exh(14) is true in this case.

<u>Second case:</u>

$w_0 \notin \lambda$wCOME(m,w) and $w_0 \notin \lambda$wCOME(j,w)

max(λwCOME(m,w)) = $\perp$

We look at every set of worlds Q which is both a subset of λwCOME(j,w) and a superset of λwCOME(m,w). Because $w_0 \notin \lambda$wCOME(m,w) and $w_0 \notin \lambda$wCOME(j,w), for every Q, $w_0 \notin$Q. Since we look only at subsets Q which are join semilattices (i.e.

Q's which contain w_0), and there are none, the second main conjunct of exh(14) is trivially true, and exh(14) is true in this case as well.

Third case:

$w_0 \notin \lambda w COME(m,w)$ and $w_0 \in \lambda w COME(j,w)$

$max(\lambda w COME(m,w)) = \bot$

We look at every set of worlds Q which is a both a subset of $\lambda w COME(j,w)$ and a superset of $\lambda w COME(m,w)$. Because $w_0 \notin \lambda w COME(m,w)$ and $w_0 \in \lambda w COME(j,w)$, for some subsets Q, $w_0 \in Q$, and for some subsets Q, $w_0 \notin Q$. We look at the Q's which are join semilattices (i.e. those who include w_0). It is clear that the sentence is false in this case, it is not the case that $w_0 \leq \bot$.

The exhaustivization of (14) is equivalent to Mary comes if and only if John comes.

Let us turn now to the clausal implicatures. A asked whether John comes. B answered that John comes, if Mary comes. Even after exhaustivization, B's answer doesn't supply A with the answer. From this fact we infer that B doesn't know if John comes. By Quality we infer that B knows that John comes if Mary comes and that Mary comes if John comes. We can safely conclude that B also doesn't know whether Mary comes.

6.2 Grice's 'holiday in France' example revisited

Our discussion of Quantity implicatures started with the following example:

<u>A holiday in France</u> (Grice 1975, 51-52)

A is planning with B an itinerary for a holiday in France. Both know that A wants to see his friend C, if to do so would not involve too great a prolongation of his journey:

A: Where does C live?

B: Somewhere in the south of France.

(Gloss: There is no reason to suppose that B is opting out [from the Cooperative Principle]; his answer is, as he well knows, less informative than is required to meet A's needs. This infringement of the first maxim of Quantity can be explained only by the supposition that B is aware that to be more informative would be to say something that infringed the maxim of Quality, 'Don't say what you lack adequate evidence for', so B implicates that he does not know in which town C lives.)

It is clear that exhaustivization won't help us in deriving the implicature in this case. All exhaustivity can do is to calculate the inference that C lives only in the South of France, and nowhere else. So, how should we analyze this example if we think that the Quantity maxim does not exist? I argue that the Quantity maxim is not needed here, what we need is just common sense.

The question *where does C live?* has a very vague interpretation. Does it inquire about an exact address? a town? a country? a continent? The context specifies the level of finegraindedness that is needed. In the context introduced by Grice, it is clear that the reply given by B does not answer A's question. So, A can justly infer that either B does not know where exactly *C* lives, or that he knows, but is unwilling to give this information. In a context where *where* refers to large areas, such as regions, countries etc. this inference will not rise.

So again, it is **Quality** that is at stake, not Quantity. In Grice's example, B's answer is **not** an answer to the question. Hence we use common sense to derive conclusions from that. We do **not** use a maxim of Quantity to compare the information content of statements uttered with statements not uttered. Such a maxim is not needed.

To sum up, we stipulate a maxim of **Quality** for question answering: "Answer the question!", and a semantic operator of **exhaustivity**. The maxim of Quality for question answering is in the spirit of Hintikka (1986) who notes that the Gricean maxims of Quantity, Quality and Relation apply most naturally and most directly to answers to questions. Exhaustivity is a device which makes it easier for speakers to comply with the maxim of Quality for question answering. It strengthens the answers given in a certain way, and it is responsible for the inferences which were labeled 'strong quantity implicatures' or 'scalar implicatures'. In other words: Quality requires the answerer to give a complete answer to the question. When the statement uttered doesn't all by itself, the questioner can assume that the answerer meant a strengthened version of the statement uttered which **does** provide a complete answer, if such a strengthening with *exh* is readily available. If even this doesn't help, the questioner must assume that the answer violates Quality. Unlike for **statements**, this doesn’t make the answerer automatically uncooperative. It only means that she doesn’t give a qualitatively acceptable answer. The questioner can infer in this case that the answerer doesn’t **know** the (complete) answer, and her statement is either interpreted as an expression of that fact (hence the ‘weak Quantity implicatures’), or as an expression of what she **does** know, in other words: a complete answer to a different question (my adviser reports that this was a well established strategy during the many oral exams that he was subjected to during his studies with Jeroen Groenendijk and Martin Stokhof).

Chapter 7

Some Notes on Further Issues Related to Exhaustivity and Scalar Implicatures

7.1 Exhaustivity in pair-list answers

Until now we have discussed exhaustiveness in one-place constituent questions and yes/no questions. But exhaustiveness shows up also in two-place constituent questions, such as in (1).

(1) Who kissed who?

Sarah and John (kissed) Bill; Bill (kissed) John

On the exhaustive interpretation, the answer in (1) states that the kissing relation includes only the following three pairs: <Sarah, Bill>, <John, Bill>, <Bill, John>.

I will here briefly suggest how the framework can be extended to these cases. I follow Groenendijk and Stokhof (1984b), and assume that the abstract of an n-place constituent question is an n-place relation, and that a 'single' short answer to such a question is an n-tuple of terms (for example, <Sarah and John, Bill> or <Bill, John>).

I take it that the answer *Sarah and John – Bill; Bill – John* is a **list** of two partial answers. I assume that exhaustivization is done globally on the whole list. I.e.

exhaustivization in (1) **adds** the information that the sum of kissers is Sarah, John and Bill, and that the sum of 'kissees' is Bill and John.

Before analyzing (1) in detail let me define a notion of summing a set of pairs.

(2) Let $T = \{<p_1, q_1>, < p_2, q_2>, \ldots. <p_n,q_n>\}$. $\Sigma T = <\sqcup\{p_1,..p_n), \sqcup\{q_1,..q_n)>$

(3) $<p_1, q_1> \sqsubseteq <p_2, q_2>$ iff $p_1 \sqsubseteq p_2$ and $q_1 \sqsubseteq q_2$

(4) Let P and Q be variables ranging over sets of the form $*X\times*X$ for some $X\subseteq ATOM$.

Let $t_1,\ldots t_n$ be a list of n pairs of terms. We associate with each t_i two interpretations. t_{ARG} of type $e\times e$ and t_{PRED} of type $<e,<e,t>>$.

Let T_i be variables on pairs of type $(e\times e)\times(<e,<e,t>>)$.

If $\alpha\in EXP_{(e\times e)\times(<e,<e,t>>)}$ and $[\![\alpha]\!] = <\mathcal{T},\mathcal{P}>$, then $[\![\alpha^1]\!] = \mathcal{T}$ and $[\![\alpha^2]\!] = \mathcal{P}$.

$$exh = \lambda T_1\ldots\lambda T_n\lambda P[P(T_1^1)\wedge\ldots\wedge P(T_n^1)\wedge\forall Q[[Q(T_1^1)\wedge\ldots\wedge Q(T_n^1)\wedge Q\subseteq P]$$
$$\rightarrow \Sigma Q \sqsubseteq \Sigma((T_1^2\cup\ldots\cup T_n^2)\cap P)]$$

(Again, I don't generalize the definition to generalized quantifiers here.)

Let us now analyze example (1):

(1) Who kissed who?

Sarah and John (kissed) Bill; Bill (kissed) John

$P = \text{ABS(who kissed who?)} \rightsquigarrow \lambda x \lambda y {*}\text{KISS}(x,y)$

$T_1^1 = \text{sarah and john, bill} \rightsquigarrow \langle s \sqcup j, b \rangle$

$T_1^2 = \text{sarah and john, bill} \rightsquigarrow \{\langle s \sqcup j, b \rangle\}$

$T_2^1 = \text{bill, john} \rightsquigarrow \langle b, j \rangle$

$T_2^2 = \text{bill, john} = \{\langle b, j \rangle\}$

$P(T_1^1) = {*}\text{KISS}(s \sqcup j, b)$

$P(T_2^1) = {*}\text{KISS}(b, j)$

$T_1^2 \cup T_2^2 = \{\langle s \sqcup j, b \rangle\} \cup \{\langle b, j \rangle\} = \{\langle s \sqcup j, b \rangle, \langle b, j \rangle\}$

Since $\{\langle s \sqcup j, b \rangle, \langle b, j \rangle\} \subseteq \lambda x \lambda y {*}\text{KISS}(x,y)$,

$(T_1^2 \cup \ldots \cup T_n^2) \cap P = \{\langle s \sqcup j, b \rangle, \langle b, j \rangle\}$

$\Sigma((T_1^2 \cup \ldots \cup T_n^2) \cap P) = \langle s \sqcup j \sqcup b, b \sqcup j \rangle$

$\text{Exh}(1) = [{*}\text{KISS}(s \sqcup j, b) \wedge {*}\text{KISS}(b, j)] \wedge$

$$\forall Q[[Q(s \sqcup j, b) \wedge Q(b, j) \wedge Q \subseteq \lambda x \lambda y {*}\text{KISS}(x,y)] \rightarrow \Sigma Q \sqsubseteq \langle s \sqcup j \sqcup b, b \sqcup j \rangle$$

In words: Sarah and John kissed Bill and Bill kissed John, and for every subset of the kissing relation that includes Sarah, John and Bill as kissers and Bill and John as kissees, the sum of kissers is a part of Sarah, John and Bill, and the sum of kissees is a part of Bill and John.

Exh(1) means that Sarah and John kissed Bill, Bill kissed John and no one else kissed anyone.

Let us do now a more complex example:

(5) How many babies did each politician kiss?

Sharon – two, Bibi – seven

The question in (5) is not a two place constituent question, however it has a pair list reading, and exhaustivization works much the same as in the previous example.
The answer in (5), when understood exhaustively, means that Sharon kissed exactly 2 babies, Bibi kissed exactly seven babies, and no other politician kissed babies. Again, I assume that exhaustivization is global on the list, and **adds** the information that the sum of politician baby kissers is Sharon and Bibi, and that at most 9 (2+7) babies were kissed by a politician.

Example (5) involves pairs of individuals and numerals. I assume that the Σ operation on numerals is summing.

I do not know how exactly the pair-list reading of the question in (5) should be analyzed. For simplicity, I'll assume it is equivalent to a two place constituent question i.e. as if there was a *which* instead of the *each* (Groenendijk and Stokhof 1984b argue for such an analysis).

$P = \text{ABS(How many babies did each politician kiss?)} \leadsto$

$\lambda x \lambda n[{*}\text{POLITICIAN}(x) \wedge |\lambda y[{*}\text{BABY}(y) \wedge {*}\text{KISS}(x,y)]| \geq n]$

$T_1^1 = \text{sharon, two} \leadsto \langle s, 2 \rangle$

$T_1^2 = \text{sharon, two} \leadsto \{\langle s, 2 \rangle\}$

$T_2^1 = \text{bibi, seven} \leadsto \langle b, 7 \rangle$

$T_2^2 = \text{bibi, seven} = \{\langle b, 7 \rangle\}$

$P(T_1^1) = \lambda x \lambda n[{*}\text{POLITICIAN}(x) \wedge |\lambda y[{*}\text{BABY}(y) \wedge {*}\text{KISS}(x,y)]| \geq n]\langle s, 2 \rangle =$

${*}\text{POLITICIAN}(s) \wedge |\lambda y[{*}\text{BABY}(y) \wedge {*}\text{KISS}(s,y)]| \geq 2$

$P(T_2^1) = {*}\text{POLITICIAN}(b) \wedge |\lambda y[{*}\text{BABY}(y) \wedge {*}\text{KISS}(b,y)]| \geq 7$

$T_1^2 \cup T_2^2 = \{\langle s, 2 \rangle\} \cup \{\langle b, 7 \rangle\} = \{\langle s, 2 \rangle, \langle b, 7 \rangle\}$

Since $\{\langle s, 2 \rangle, \langle b, 7 \rangle\} \subseteq P$,

$(T_1^2 \cup \ldots \cup T_n^2) \cap P = \{\langle s, 2 \rangle, \langle b, 7 \rangle\}$

$\Sigma((T_1^2 \cup \ldots \cup T_n^2) \cap P) = \langle s \sqcup b, 9 \rangle$

$$\begin{aligned}
\text{Exh}(5) = {} & [{*}\text{POLITICIAN}(s) \wedge |\lambda y[{*}\text{BABY}(y) \wedge {*}\text{KISS}(s,y)]| \geq 2 \wedge \\
& {*}\text{POLITICIAN}(b) \wedge |\lambda y[{*}\text{BABY}(y) \wedge {*}\text{KISS}(b,y)]| \geq 7] \wedge \\
& \forall Q[[Q(\langle s,2 \rangle) \wedge Q(\langle b,7 \rangle) \wedge Q \subseteq \lambda x \lambda n[{*}\text{POLITICIAN}(x) \wedge \\
& |\lambda y[{*}\text{BABY}(y) \wedge {*}\text{KISS}(x,y)]| \geq n] \rightarrow \Sigma Q \sqsubseteq \langle s \sqcup b, 9 \rangle
\end{aligned}$$

In words: Sharon is a politician that kissed at least two babies, and Bibi is a politician that kissed at least seven babies, and for every subset of pairs consisting of a politician and a number of babies kissed by him/her and which includes the pairs <Sharon, 2>

and <Bibi, 7>, the sum of the politicians is Sharon and Bibi and the sum of babies kissed is 9.

Exh(5) means that Sharon kissed exactly 2 babies and Bibi kissed exactly 7 babies, and that no other politician kissed a baby.

I leave it for further research how the formulation of *exh* in (3) should be generalized to cover more complicated cases. Consider for example (6):

(6) How many babies did each politician kiss?
Sharon and Barak – three, Bibi – seven

(6) is ambiguous between a collective and a distributive interpretation. On the collective interpretation, Sharon and Barak kiss 3 babies between them, on the distributive interpretation Sharon and Barak kiss 3 babies each. The exhaustivization of the collective interpretation works the same as in example (5); the number of babies kissed is at most 10. Whereas on the exhaustive interpretation of the distributive case the number of babies kissed is at most 13 (the summing of babies is not 3+7, but 3×2+7)

7.2 Exhaustivity in questions

Not only declarative sentences have implicatures. The question in (7) can be understood as asking who has **exactly** 2 children:

(7) Who has $[\text{two}]_F$ children?

No one. Everyone has either less than two or more than two.

If scalar implicatures are exhaustiveness effects, and if exhaustiveness is an answerhood constraint, then how come questions can have scalar implicatures?

I will here tentatively sketch an approach to this phenomenon. According to 'alternative semantics' theories of focus (see Rooth 1985, 1992, 1996), declarative sentences have, besides their ordinary semantic values, also "focus semantic values". The focus semantic value is the set of all propositions obtained by replacing each focus with alternatives of the same type. Kadmon (2001) argues that not only declarative sentences, but also questions have focus semantic values. For example, the focus on *two* in (7) signals that the speaker is choosing the question "*who has two children?*" out of a set of alternative questions, questions of the form "*who has n children?*"

Roberts (1996b) sees focus as essentially a discourse regulating device (see also Schwarzschild 1999). On Roberts theory, the representation of the context includes not only the common ground but also the collection of questions that are assumed to be under discussion, and have not yet been answered. Roberts assumes that the general constraint on the use of focus is that the focus semantic value of a declarative sentence must be identical to the ordinary semantic value of the last question under discussion (QUD).

Roberts assumes that each utterance in the discourse (be that a statement or a question) is related to a preceding utterance. If it is a statement, it must give a partial answer to the last QUD. If it is a question, it must be a sub-question of the last QUD (i.e. a complete answer to it contextually entails a partial answer to the last QUD).

It may be that exhaustivity is triggered not only by the question/answer pairs, but also by question/sub-question pairs. The question in (7), *Who has [two]$_F$ children?* is a sub-question of *Who has how many children?*

Essentially, I analyze this case as a two place constituent question which is ‘answered’ by a one place constituent question.

(8) Who has how many children?

Who has [two]$_F$ children?

I assume that *exh* in this case is a relation between the abstract of a two-place constituent question and the ‘short answer’ <*who, two*>. I take it that *who* is interpreted as a free variable, *x*, which will be bound by a λ operator, from the “outside”.

$$exh = \lambda x \lambda T \lambda P[P(T^1) \wedge \forall Q[[Q(T^1) \wedge Q \subseteq P] \rightarrow \Sigma Q \sqsubseteq \Sigma(T^2 \cap P)]$$

P= ABS(who has how many children?)$\rightsquigarrow \lambda x \lambda n |\lambda y[{}^*CHILD(y) \wedge {}^*HAVE(x,y)]| \geq n$

T^1= <who, two> $\rightsquigarrow \langle x, 2\rangle$

T^2= <who, two> $\rightsquigarrow \{\langle x, 2\rangle\}$

$P(T^1) = [|\lambda y[*CHILD(y) \wedge *HAVE(x,y)]| \geq 2]$

Since $\{<x, 2>\} \subseteq P$,

$T^2 \cap P = \{<x,2>\}$ and $\Sigma(T^2 \cap P) = <x,2>$

$$exh(8) = \lambda x\{|\lambda y[*CHILD(y) \wedge *HAVE(x,y)]| \geq 2 \wedge$$
$$\forall Q[[Q(<x,2>) \wedge Q \subseteq \lambda x \lambda n|\lambda y[*CHILD(y) \wedge *HAVE(x,y)]| \geq n] \rightarrow$$
$$\Sigma Q \leq <x, 2>]\}$$

In words: the set of x's such that: x has (at least) 2 children, and for every subset Q of pairs consisting of x and a number of children (s)he has, and which includes the pair <x, 2>, the sum of Q is part of x and the sum of the number of children is smaller or equal to 2.

Exh(8) is the set {x: x has exactly 2 children}, which is the abstract of the question *who has exactly 2 children?*

7.3 Scalar implicatures out of the blue

Consider the following sentences:

(9) Mary has 3 children

(10) Mary has exactly 3 children

(11) Mary has at least 3 children

(12) Mary has at most 3 children

Standard introduction to semantics and/or pragmatics text books will probably tell you that although (9) conveys (10) in most contexts, it means (11), and conversationally implicates (12).

According to the theory of implicatures sketched in this dissertation, a sentence does not simply have a scalar implicature. (9) may or may not be interpreted as (10), and this (largely) depends on the question it answers. For example, if we exhaustivize (9) relative to the question *how many children does Mary have?* we'll get the interpretation that Mary has exactly 3 children. If we exhaustivize (9) relative to the question *who has 3 children?* We'll get the interpretation that only Mary has 3 children, and if we exhaustivize (9) relative to *Does Mary have 3 children?* exhaustivization would not add anything. But what if (9) does not answer any question? What if (9) is the first utterance in discourse? Consider, for example (13).

(13) Mary has 3 children. They are cute.

Kadmon (2001) argues that in order to satisfy the uniqueness condition on the definite pronoun, we accommodate a scalar implicature, even in contexts were such an implicature is not independently triggered. How can we interpret this fact in our theory?

We may adopt a radical underspecification theory of meaning. Let us associate with each declarative sentence not only one meaning, but a set of meanings – the set of all possible exhaustivizations of that sentence. I.e. the meaning of a sentence is determined by the set of questions which it can answer. As the discourse continues, we may get rid of meanings which are no longer compatible with the information accumulated . We may analyze (13) as follows. Initially, we associate with (13) (at least) 4 interpretation: Mary has 3 children, Only [Mary] has at least 3 children, Mary has only [3] children, Mary has only [3 children]. The definite pronoun *they* requires a unique reference, hence we end up with the interpretations that put 3 in the scope of *only*. Of course, more can be said about this.

Chapter 8

Conclusion

The ‘prime directive’ of the researches in the Gricean tradition, has mostly been: “Keep your semantics extremely thin, and try as hard as you can to explain as much as you **think you can** with (your own version of) Grice’s conversational maxims” (see for example, Harnish 1976, Sadock 1978, 1981, 1984, Atlas and Levinson 1981, Horn 1984). This approach is sometimes called *radical pragmatics* (although, as Levinson 1983 remarks, the term *radical semantics* might be more appropriate). Horn’s theory of scalar implicatures is a model example for *radical pragmaticists*. I have argued, in the bulk of this dissertation, that such an approach to scalar implicatures is wrong, and that the strong interpretations attributed to the application of some version of Grice’s first maxim of Quantity, are available through the semantic operation of exhaustivization.

We have replaced the Grice/Horn theory of Quantity implicatures with a theory which consists of:

1. A strong maxim of Quality for questions: “Answer the question!”
2. An exhaustivization schema which exhaustivizes the meaning of a statement relative to a question.

We have made the assumption that exhaustivization is generally available and can be used as a strategy to satisfy the strong Quality maxim for questions.

I have shown this theory to be superior to the neo-Gricean theories of scalar implicatures (Horn 1972, 1984, 1989, Gazdar 1979, Atlas and Levinson 1981, Levinson 1983, 2000, Gamut 1991, Matsumoto 1995) in several ways:

1. We do without the maxim of Quantity which over-generates wildly non existing implicatures.
2. We do not need to stipulate a strengthening process for 'weak quantity implicatures'.
3. We do not need to stipulate Horn scales such as <Mary, John and Mary> and <or, and>, and then explain away why scales such as <Mary, only Mary> and <or, exclusive or> don't exist. The semantics of the exhaustivity operator refers to domains which form join semilattices, and this restriction explains why we find the Horn scales we find, when we find them.
4. The facts about implicature inheritance and suspension/cancellation ('the projection problem for scalar implicatures') come out naturally within our theory.

I have also shown that the 'weak Quantity implicatures' (implicatures about the lack of knowledge of the answerer) can be derived without reference to the Quantity maxim.

Exhaustivization had its start in Groenendijk and Stokhof's (Groenendijk and Stokhof 1984b) theory of questions and their operation of *exh*. I have shown that my operation is superior to theirs:

1. My operation works also for plural NP's (not only for singular NP's).
2. My operation naturally generalizes to ordered domains that Groenendijk and Stokhof did not cover.

Unlike Groenendijk and Stokhof, I do not assume that *exh* is a part of the meaning of a statement **as** an answer to the question. I assume that *exh* is an operation which is **triggered** by the question/answer relation due to the strong maxim of Quality for questions.

References

Atlas, Jay David and Stephen Levinson 1981: "*It*-Clefts, Informativeness, and Logical Form: Radical Pragmatics (Revised Standard Version)." In Peter Cole (ed.), R*adical Pragmatics*, Academic Press, New York, 1-61.

Beaver, David 1995: *Presupposition and Assertion in Dynamic Semantics*. Ph.D. dissertation, University of Edinburgh.

Bonomi, Andrea and Paolo Casalegno 1993: "*Only*: Association with Focus in Event Semantics." *Natural Language Semantics* 2.1, 1-45

Chierchia, Gennaro (ms.): "Scalar Implicatures, Polarity Phenomena, and the Syntax/Pragmatics Interface." University of Milan, May 2001.

Chierchia, Gennaro and Sally McConnell-Ginet 2000: *Meaning and Grammar: An Introduction to Semantics*, second edition, MIT Press, Cambridge MA.

Cresswell, Maxwell J. 1973: *Logics and Languages*, Methuen, London.

Frege, Gottlob 1892: "Über Sinn und Bedeutung." *Zeitschrift für Philosophie und Philosophische Kritik* 100, 25-50. Translation by Max Black: "On Sense and Reference." The philosophical Review 57: 207-230 (1948). Reprinted in Peter Ludlow (ed.), 1997, Readings in the Philosophy of Language, MIT Press, Cambridge, MA, 563-583.

von Fintel, Kai 1994: *Restrictions on Quantifier Domains*. Ph.D. dissertaion, GSLA, UMass, Amherst.

Gamut, L.T.F 1991: *Logic, Language, and Meaning. Volume 1: Introduction to Logic*. The University of Chicago Press, Chicago.

Gazdar, Gerald 1979: *Pragmatics: Implicature, Presupposition, and Logical Form*, Academic Press, New York.

Groenendijk, Jeroen and Martin Stokhof, 1984a: "On the Semantics of Questions and the Pragmatics of Answers." In Fred Landman and Frank Veltman (eds.), *Varieties of Formal Semantics*, Foris, Dordrecht, 143-170.

Groenendijk, Jeroen and Martin Stokhof, 1984b: *Studies on the Semantics of Questions and the Pragmatics of Answers*, Ph.D. dissertation, University of Amsterdam.

Groenendijk, Jeroen and Martin Stokhof, 1989: "Type-Shifting Rules and the Semantics of Interrogatives." In Gennaro Chierchia, Barbara H. Partee and Raymond Turner (eds.), *Properties, Types and Meaning, Volume II: Semantic Issues*, Kluwer, Dordrecht, 21-68. Reprinted in Paul Portner and Barbara H. Partee (eds.), 2002, *Formal Semantics: The Essential Readings*, Blackwell, Oxford.

Grice, Paul 1975: "Logic and Conversation." In Peter Cole and Jerry L. Morgan (eds.), *Syntax and Semantics 3: Speech Acts*, Academic Press, New York, 41-58. Reprinted in Paul Grice, *Studies in the Way of Words*, Harvard University Press, Cambridge, MA, 22-40, and in Kasher, Asa (ed.) 1998: *Pragmatics: Critical Concepts, Volume 4: Presupposition, Implicature and Indirect Speech Acts*, Routledge, London, 145-161.

Grice, Paul 1989: "Further Notes on Logic and Conversation." In Paul Grice, *Studies in the Way of Words*, Harvard University Press, Cambridge, MA, 41-57. Reprinted in Kasher, Asa (ed.) 1998: *Pragmatics: Critical Concepts, Volume 4: Presupposition, Implicature and Indirect Speech Acts*, Routledge, London, 162-176.

Harnish, Robert M. 1976: "Logical Form and Implicature." In T. Bever, J. Katz, and T. Langendoen (eds.) An Integrated Theory of Linguistic Ability, Crowell, New York, 464-479. Reprinted in Reprinted in Kasher, Asa (ed.) 1998: *Pragmatics: Critical Concepts, Volume 4: Presupposition, Implicature and Indirect Speech Acts*, Routledge, London, 230-314.

Heim, Irene 1983: "On the Projection Problem for Presuppositions." In M. Barlow, D. Flickinger and M. Wescoat (eds.), *Proceedings of WCCFL 2,* Stanford University, 114-125. Reprinted in Paul Portner and Barbara H. Partee (eds.), 2002, *Formal Semantics: The Essential Readings*, Blackwell, Oxford.

Hintikka, Jaakko 1986: "Logic of Conversation as a Logic of Dialogue." In *Philosophical Grounds of rationality, Intentions, categories and Ends*, Clarendon Press, Oxford, 81-114. Reprinted in Kasher, Asa (ed.) 1998: *Pragmatics: Critical Concepts, Volume 4: Presupposition, Implicature and Indirect Speech Acts*, Routledge, London, 419-435.

Hirschberg, Julia Bell 1985: *A Theory of Scalar Implicature*, Ph.D. dissertation, University of Pennsylvania.

Horn, Laurence R. 1972: *On the Semantic Properties of Logical Operators in English*, Ph.D. dissertation, UCLA. Reproduced in 1976 by the Indiana University Linguistics Club, Bloomington.

Horn, Laurence R. 1984: "Towards a New Taxonomy for Pragmatic Inference: Q-based and R-based Implicature." In D. Schiffrin (ed.), *Meaning, Form and use in Context*, Georgetown University Press, Washington DC, 11-42. Reprinted in Kasher, Asa (ed.) 1998: *Pragmatics: Critical Concepts, Volume 4: Presupposition, Implicature and Indirect Speech Acts*, Routledge, London, 383-418.

Horn, Laurence R. 1988: “Pragmatic Theory.” In Frederick J. Newmeyer (ed.), *Linguistics: The Cambridge Survey, Volume 1, Linguistic Theory: Foundations*, Cambridge University Press, Cambridge, 113-145.

Horn, Laurence R. 1989: *A Natural History of Negation*, University of Chicago Press, Chicago. Reissued in 2001 by CSLI Publications, Stanford.

Jacobs, Joachim 1983: *Fokus und Skalen: Zur Syntax und Semantik von Gradpartikeln im Deutschen*, Niemeyer, Tübingen.

Kadmon, Nirit 2001: *Formal Pragmatics: Semantics, Pragmatics, Presupposition and Focus*, Blackwell, Oxford.

Karttunen, Lauri 1974: “Presuppositions and Linguistic Context.” *Theoretical Linguistics* 1, 181-194. Reprinted in Kasher, Asa (ed.) 1998: *Pragmatics: Critical Concepts. Volume 4: Presupposition, Implicature and Indirect Speech Acts*, Routledge, London, 32-46.

Karttunen, Lauri and Stanley Peters 1979: “Conventional Implicature.” In Ch. Oh and D. Dinneen (eds.), *Syntax and Semantics 11: Presupposition*, Academic Press, New York, 1-56.

Klein, Wolfgang and Arnim von Stechow 1982: “Intonation und Bedeutung von Fokus.”, Arbeitspapier des SFB 99, No. 77, Konstanz.

Kratzer, Angelika 1991: “The Representation of Focus.” In Arnim von Stechow and Dieter Wunderlich (eds.), *Semantik/Semantics: An International Handbook of Contemporary Research*, de Gruyter, Berlin.

Krifka, Manfred 1991: “A Compositional Semantics for Multiple Focus Constructions.” In S. Moore and A. Z. Wyner (eds.), *Proceedings of SALT 1*, Cornell University Working Papers in Linguistics No. 10, 127-158.

Ladusaw, William A. 1979: *Polarity Sensitivity as Inherent Scope Relations*, Ph.D. dissertation, University of Texas, Austin. Reproduced in 1980 by the Indiana University Linguistics Club, Bloomington.

Landman, Fred 1996: “Plurality.” In Shalom Lappin (ed.), *The Handbook of Contemporary Semantic Theory*, Blackwell, Oxford, 425-457.

Landman, Fred 2000: *Events and Plurality: The Jerusalem Lectures*, Kluwer, Dordrecht.

Landman, Fred 2004: *Indefinites and the Type of Sets*, Blackwell, Oxford.

Levinson, Stephen C. 1983: *Pragmatics*, Cambridge University Press, Cambridge.

Levinson, Stephen C. 2000: *Presumptive Meanings: The Theory of generalized Conversational Implicature*, MIT Press, Cambridge, MA.

Lewis, David 1979: “Scorekeeping in a Language Game.” In R. Bäuerle, U. Egli, and A. von Stechow (eds.), *Semantics from Different Points of View*, Springer, Berlin 172-187. Reprinted in Paul Portner and Barbara H. Partee (eds.), 2002, *Formal Semantics: The Essential Readings*, Blackwell, Oxford.

Link, Godehard 1983: “The Logical Analysis of Plurals and Mass Terms: A Lattice-theoretic Approach.” In R. Bäuerle, Ch. Schwarze and A. von Stechow (eds.), *Meaning, Use, and Interpretation of Language*, de Gruyter, Berlin. Reprinted in Paul Portner and Barbara H. Partee (eds.), 2002, *Formal Semantics: The Essential Readings*, Blackwell, Oxford.

Matsumoto, Yo 1995: “The Conversational Condition on Horn Scales.”, *Linguistics and Philosophy* 18.1, 21–60.

Partee, Barbara H. 1987: “Noun Phrase Interpretation and Type Shifting Principles.” In Jeroen Groenendijk, Dick de Jongh, and Martin Stokhof (eds.), *Studies in Discourse Representation Theory and the Theory of Generalized Quantifiers*, Foris, Dordrecht, 115-143. Reprinted in Paul Portner and Barbara H. Partee (eds.), 2002, *Formal Semantics: The Essential Readings*, Blackwell, Oxford.

Potts, Chris 2005: *The Logic of Conventional Implicatures*. Oxford Studies in Theoretical Linguistics. Oxford University Press, Oxford.

Roberts, Craige 1989: “Modal Subordination and Pronominal Anaphora in Discourse.” Linguistics and Philosophy 12, 683-721.

Roberts, Craige 1996a: “Anaphora in Intensional Contexts.” In Shalom Lappin (ed.), *Handbook of Contemporary Semantics*, Blackwell, Oxford, 215-246.

Roberts, Craige 1996b: “Information Structure in Discourse: Towards an Integrated Formal Theory of Pragmatics.” In J. Hak Yoon and A. Kathol (eds.), *Ohio State University Working Papers in Linguistics, Volume 49*.

Rooth , Mats 1985: *Association with Focus*, Ph.D. dissertation, University of Massachusetts, Amherst. Reproduced by the GLSA, Department of Linguistics, University of Massachusetts, Amherst.

Rooth, Mats 1992: “A Theory of Focus Interpretation.” *Natural Language Semantics* 1.1, 75-116.

Rooth, Mats 1996: “Focus.” In Shalom Lappin (ed.), *The Handbook of Contemporary Semantic Theory*, Blackwell, Oxford, 271-297.

Rullmann, Hotze 1985: *Maximality in the Semantics of wh-Constructions*, Ph.D. dissertation, University of Massachusetts, Amherst. Reproduced by the GLSA, Department of Linguistics, University of Massachusetts, Amherst.

Sadock, Jerrold M.1978: "On Testing for Conversational implicature." In P. Cole (ed.), S*yntax and Semantics 9: Pragmatics*, Academic Press, New York. 281-297. Reprinted in Kasher, Asa (ed.) 1998: *Pragmatics: Critical Concepts. Volume 4: Presupposition, Implicature and Indirect Speech Acts*, Routledge, London, 315-321.

Sadock, Jerrold M. 1981: "Almost.". In P. Cole (ed.), *Radical Pragmatics*, Academic Press New York, 257-271.

Sadock, Jerrold M. 1984: "Whiter Radical Pragmatics?." In D. Schiffrin (ed.), *Meaning, Form and use in Context*, Georgetown University Press, Washington DC, 139-149.

Scha, Remko 1981: "Distributive, Collective and Cumulative Quantification." In Jeroen Groenendijk, Theo Janssen and martin Stokhof (eds.), *Formal Methods in the Study of Language*, Mathematical Center Tracts, Amsterdam. Reprinted in Jeroen Groenendijk, Theo Janssen and martin Stokhof (eds.), 1984, *Truth, Interpretation, and Information*, Foris, Dordrecht, 131-158.

Schwarzschild, Roger 1999: "GIVENness, AvoidF and Other Constraints on the Placement of Accent." *Natural Language Semantics*, 7.2, 141-177.

Schwarzschild, Roger and Karina Wilkinson 2002: "Quantifiers in Comparatives: A Semantics of Degree based on Intervals. *Natural Language Semantics*, 10.1 1-41.

Sharvy, Richard 1980: "A More General Theory of Definite Descriptions." *The Philosophical Review* 89.

Simons, Mandy 2001: "On the Conversational Basis of Some Presuppositions." In R. Hastings, B. Jackson and Z. Zvolensky, (eds.) *Proceedings of Semantics and Linguistic Theory* 11, CLC Publications, Ithaca, NY.

Stalnaker, Robert C. 1974: "Pragmatic Presuppositions." In M.K. Munitz and D.K. Unger (eds.), *Semantics and Philosophy*, New York University Press, New York, 97-213. Reprinted in Kasher, Asa (ed.) 1998: *Pragmatics: Critical Concepts, Volume 4: Presupposition, Implicature and Indirect Speech Acts*, Routledge, London, 16-31, and in Stalnaker, Robert 1999: *Context and Content: Essays on Intentionality in Speech and Thought*, Oxford University Press, Oxford, 47-62.

Stalnaker, Robert C. 1975: "Assertion." In Asa Kasher (ed.), *Language in Focus*, Reidel. Reprinted in Stalnaker, Robert 1999: *Context and Content: Essays on Intentionality in Speech and Thought*, Oxford University Press, Oxford, 63-77, and in Paul Portner and Barbara H. Partee (eds.), 2002, *Formal Semantics: The Essential Readings*, Blackwell, Oxford.

Stalnaker, Robert C. 1978: "Pragmatic Presuppositions." In Peter Cole (ed.), *Syntax and Semantics 9: Pragmatics*, Academic Press, New York, 315-332. Reprinted in Stalnaker, Robert 1999: *Context and Content: Essays on Intentionality in Speech and Thought*, Oxford University Press, Oxford, 78-95.

von Stechow, Arnim 1981: “Topic, Focus and Local Relevance.” In W. Klein and W. Levelt (eds.), *Crossing the Boundries in Linguistics*, Reidel, Dordrecht.

von Stechow, Arnim 1989: “Focusing and Backgrounding Operators.” Technical Report 6, Fachgruppe Sprachwissenschaft, Universität Konstanz.

Westerståhl, Dag 1984: “Determiners and Context Sets.” In Johan van Benthem and Alice ter Meulen, (eds.), *Generalized Quantifiers in Natural Language*, Foris, Dordrecht, 45-71.

Winter, Yoad 1998: *Flexible Boolean Semantics*. Ph.D. Dissertation, Utrecht University.